

**OPERATIONAL PROBLEMS IN PUMPING NON-SETTLING SLURRIES RESOLVED  
USING AN IMPROVED LAMINAR FLOW PIPE FITTING LOSS MODEL**

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**ABSTRACT**

A case history is presented pertaining to five pumping systems that operated satisfactorily until a new production requirement was imposed on the pumping systems. A new slurry product initially developed at lab scale was introduced into the production plant for an initial trial run. Problems began to surface immediately on three out of five batch process

pumping systems when the slurry could not be pumped through the plant at contract rate. Additionally, significant "heels" (unwanted fluid levels) were left in some of the suction vessels that were unable to be pumped out, resulting in considerable yield losses. This manufacturing problem had not been anticipated by the team, and without quick resolution, a loss of customer confidence and a significant delay in the new product would have resulted.

Investigation and analysis of the system revealed two major problem areas in pumping non-settling slurries in laminar regimes:

- Initial prediction of head losses through suction piping fittings was flawed using traditional hydraulic loss methods. The original calculations for  $NPSH_A$  values for the pumps predicted adequate NPSH margin. The fluid was non-Newtonian and was operating in the laminar regime. Upon further investigation, a weakness was revealed in predictions of fitting losses for laminar flow through the pipe fittings. An improved model for predicting losses through pipe fittings was identified and implemented. The improved model matched operational data much better and provided the critical insight needed to resolve the operational problems and get the facility operating.
- Piping arrangements that allow for features such as clean-out ports (e.g., branch flow tees) can be counterproductive to unrestricted flow of the process liquid in systems with non-settling slurries operating in laminar regimes. Tees, elbows, diameter changes, and other fittings can introduce significant head losses in the pumping system.

The authors present an improved method for analyzing fitting losses in pumping systems when dealing with non-settling slurries operating in the laminar regime. In addition, design considerations are presented to minimize the impact that piping has on the pumping system when handling non-settling slurries operating in the laminar regime.

**INTRODUCTION**

*Description of Pumping Systems*

Five batch pumping systems were in place in an existing

process plant. In each pumping system, the process liquid is fed into the pump from a large suction vessel which contains a mixer to keep the liquid in a sheared state. The pressure in the vapor space of the suction vessel is atmospheric. The piping from the suction vessel to the pumps is not a straight path in the systems, with some being more complicated than others. The liquid exits the pumps and goes through discharge piping. There is a minimum flow recirculation line in the discharge of each pump that is regulated by a pinch valve. Pumping systems #1, #3, #4, and #5 are transfer systems moving fluid from one tank to another and a representation is shown in Figure 1. Pumping system #2 moves the fluid to a machine which interacts with the fluid and this machine requires a minimum inlet pressure and a representation is shown in Figure 2.

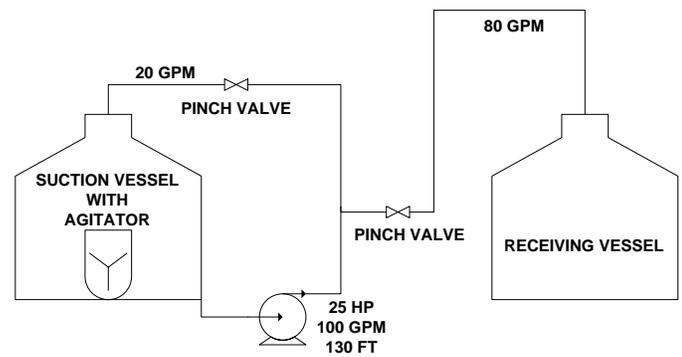


Figure 1. Representation of Pumping Systems #1, #3, #4, and #5

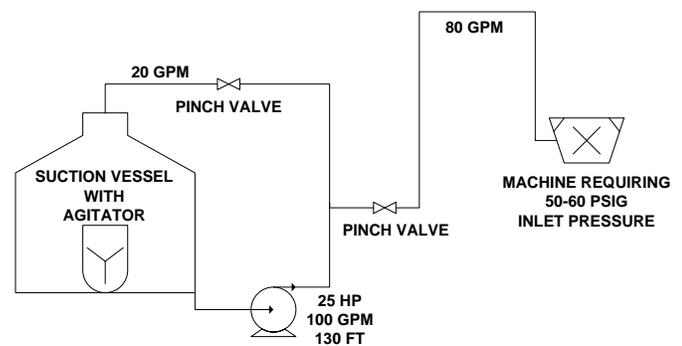


Figure 2. Representation of Pumping System #2

Newtonian fluids cover conventional fluids such as water, where the fluid shear stress is directly proportional to shear rate. The proportionality constant is the viscosity of the fluid. This relationship is observed in the solid line of Figure 3 at the left. Here it is apparent that the shear stress varies directly with shear rate. The solid line is for a Newtonian fluid.

The viscosity of a Newtonian fluid is not a function of the fluid dynamics (e.g., velocity, which is directly proportional to shear rate) or a function of time. This can be observed on the right of Figure 3, where the viscosity has no dependence on shear rate (for the solid line, which is Newtonian).

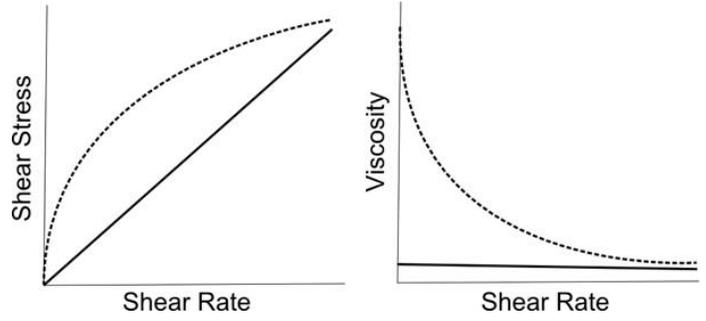


Figure 3. Steady shear rheological behavior, shown with shear stress and viscosity as a function of shear rate. Dotted line is shear thinning fluid; solid line is Newtonian fluid.

A fluid which exhibits a viscosity dependence on the fluid dynamics (e.g., velocity/shear rate) or time is referred to as non-Newtonian. A fluid in which the shear stress and shear rate follow a straight-line on a log-log plot is referred to as a power law fluid.

In practice, this means the viscosity varies for different velocities. In the case of a power law fluid, as the velocity increases, the viscosity decreases. This is also known as “shear thinning” behavior.

In this case, the pumped media is a non-settling slurry. The slurry has the characteristic of being shear thinning and acting like a power law fluid. The fluid in this case is not drilling mud, but it looks and behaves somewhat like many drilling muds.

The process followed to calculate the pressure drop for a power law fluid (and most other non-Newtonian fluids) is to first perform a rheological test on the fluid. A viscometer is used to measure the shear stress at different shear rates. Often the test is done with increasing shear rate and then decreasing shear rate to check for hysteresis. A true power law fluid will not exhibit any significant hysteresis. This data is then used to determine the power law constants.

The viscometer test was done on the fluid in this case study. The process of determining power law constants was pursued and the raw rheological data followed a power law model quite well. This confirmed the fluid was non-Newtonian in its behavior and that the slurry was non-settling. If the slurry was of a settling nature the power law model would not have fit the data. Later in this paper the mathematical details of how these constants are used to calculate pressure drop will be discussed.

Once the power law data was applied to the systems in question, it was apparent that the Reynolds number was in the laminar regime.

*Description of Pump*

The pumps used in the production facility were ASME B73.1 pumps using open impellers and dual mechanical seals. The metallurgy of each pump was adequate to handle the abrasiveness of the slurry.

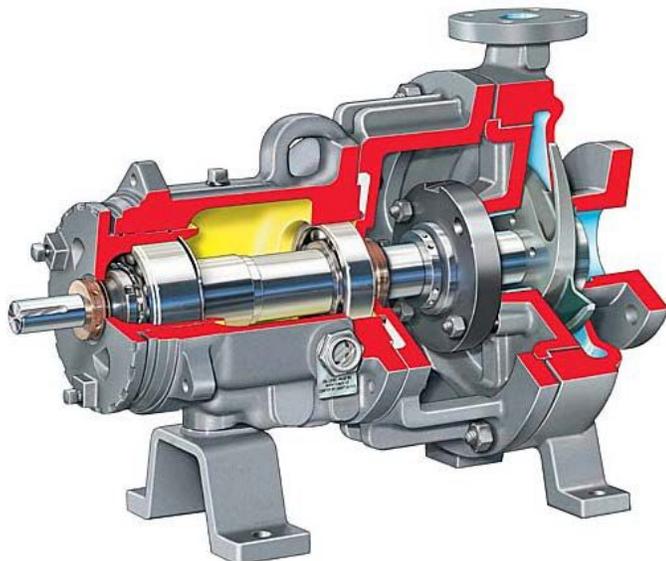


Figure 4. Cross section of pump for each system (Courtesy Flowserve Corporation)

- Flow rates were not adequate to meet production demands.
- Inadequate pressure was being delivered downstream to users requiring a certain minimum pressure.

#### Heel Left in Vessels

In three of five of the pumping systems, a significant heel was left in the vessel. For the three problem vessels, the pumps initially moved the slurry out of the vessels consistently, and the vessel levels came down steadily. However, when the vessel levels reached 3 to 5 ft (0.9 to 1.5 m), depending on the vessel, the level stopped dropping as the pump flow rapidly dropped to zero. It was not clear at this point if there was a suction issue, a pump issue, or a discharge issue.

Two of the five pumping systems did not exhibit problems, even though they were handling the same process liquid. Pumping system #3 did not have an issue. This vessel was continuously fed and the level never dropped below 5 ft (1.9 m) above the centerline of the pump. Pumping system #5 did not leave a heel in the vessel.

#### Inadequate Flow and Pressure Delivered

For the five systems, three had issues with meeting flows or downstream pressure requirements. Of the three problematic systems, two had flows that were not adequate to keep up with production demands. These two systems were simple transfer systems, so the only thing noticed was a flow inadequacy issue. The one remaining problematic system required the pump to feed a machine downstream, and inadequate pressure was being delivered to the machine for it to function properly.

## INVESTIGATION

#### System Hydraulic Model

A commercially available software package (Applied Flow Technology, 2008) was used to model the systems to better understand the hydraulics. All five pumping systems were modeled to see if there were differences that might explain why two systems were working, while the other three were not. The modeling software was able to put in characteristics for a power law fluid. Thus, changes in shear could be accounted for properly.

The initial analysis for each pumping system showed that the pumps had enough NPSH margin to pump the tanks virtually down to a zero level. Table 1 shows the original model calculations and breaks down the  $\Delta P$  for the pipe and fittings, as well as the predicted  $NPSH_A$ ,  $NPSH_R$  and NPSH margin values. Of course, a minimal heel would still be left in any vessel due to the vortexing and eventual loss of prime that occurs when the vessel level gets too low. Based on the fact that the NPSH showed no problems in the analysis, the focus was turned towards the pump and the discharge piping.

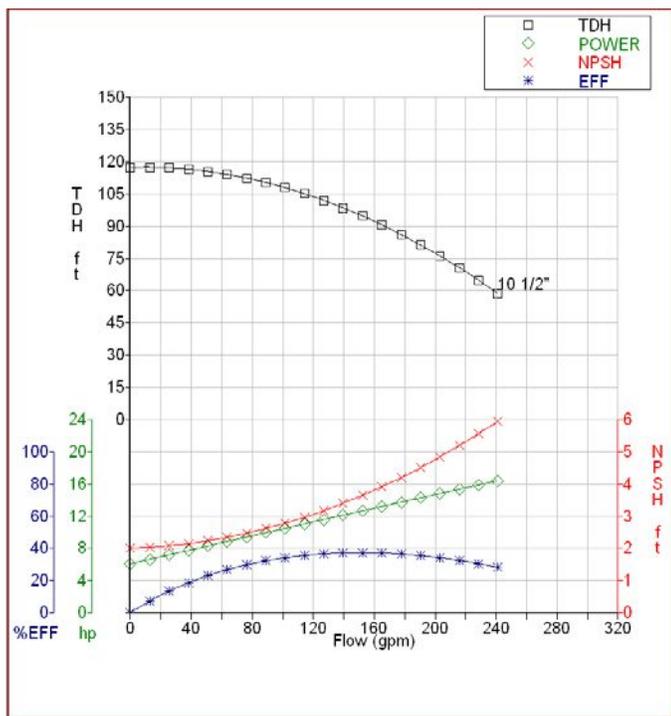


Figure 5. Representative Pump Curve at 1750 rpm (all pumps nearly identical except for changes in impeller diameter)

#### Description of Problem

Prior to the trial of the new process liquid, all five batch systems had moved liquid in a manner satisfactory to production needs. However, after the new product was introduced, three of the five pumping systems developed multiple problems:

- Significant heels of liquid from 3 to 5 ft (0.9 to 1.5 m) were left in the suction vessels when the pump would cease pumping.

Table 1. Original Hydraulic Model Predictions Using Non-Newtonian Pipe Friction and Standard K Factors for Valves and Fittings

#	Flow gpm (m <sup>3</sup> /hr)	ΔP Suction Pipe psid (kPa)	ΔP Suction Pipe Fittings psid (kPa)	NPSH <sub>A</sub> ft (m)	NPSH <sub>R</sub> ft (m)	NPSH Margin ft (m)
1	170 (38.6)	0.7 (4.8)	0.5 (3.4)	11.6 (3.5)	4.1 (1.2)	7.5 (2.3)
2	69 (15.7)	0.2 (1.4)	0.1 (0.7)	12.8 (3.9)	2.3 (0.7)	10.5 (3.2)
3	114 (25.9)	0.2 (1.4)	0.2 (1.4)	17.4 (5.3)	3.0 (0.9)	14.4 (4.4)
4	95 (21.6)	1.4 (9.7)	0.2 (1.4)	11.2 (3.4)	2.7 (0.8)	8.5 (2.6)
5	95 (21.6)	0.1 (0.7)	0.1 (0.7)	13.1 (4.0)	2.7 (0.8)	10.4 (3.2)

Based on the shear rate of the fluid in the suction, it was anticipated that a bulk viscosity of ~200 cp was entering the pump suction. In addition, the pumps were operating at around 60% of B.E.P. (Best Efficiency Point). The pumps were configured ideally for pumping a viscous liquid:

- Operating at 1750 rpm –with a viscous fluid, it is best to operate at a low speed to allow the fluid to more easily flow into the pump and keep up with feeding the pump.
- Operating to the left of B.E.P. – centrifugal pump performance (e.g., flow, head, power) is less impacted with a pump operating to the left of B.E.P. when pumping a viscous liquid. There is less deviation on the flow and head produced by the pump in the viscous fluid as compared to its water performance.
- Viscosity value of 200 cp – a value of 200 cp is within the range where a centrifugal pump does not lose significant performance. There was a mixer in the suction vessel to provide initial shear to the liquid as well so it flowed more easily to the pump.

Flow data, pressure data at the pump discharge, and power data were obtained from the field. Unfortunately, there was no pressure gauge on the pump suction. At the time the pumps were pumping well, the values for flow, pressure differential across the pump, and power matched the predicted performance curve well. Thus, it was felt there was no problem with the pump performance. However, when the pumps entered the regime where they stopped pumping, flow went to zero, discharge pressure dropped off considerably, and power dropped off considerably.

The discharge piping was rather long on each of the systems. Two of the systems had an excessive number of fittings in the discharge, including many sharp branch flow tees, 45 degree bends, 90 degree bends, abrupt diameter

changes, and pinch style valves. The initial system model again predicted there should be no problem delivering the required flows and pressures needed.

The troubleshooting focus then shifted to any other points in the system where flow might be unexpectedly bypassing back to the suction vessel. Each of the pumps had a minimum recirculation flow bypass line with a manual pinch valve in the line to regulate the flow. It was found that these valves were grossly oversized. However, they were nearly shut to compensate for this. Ultrasonic flow measurements were taken and it was found that these recirculation lines were not passing more flow than expected.

So everything seemed to check out that the system should perform as expected when running the hydraulic model. Three pieces of data that did not agree with the model were the heel that was left in the vessel, the flow rates, and the pressure well downstream of the pump at the user.

More abstract theories began to surface as the troubleshooting team grew more desperate. Some wondered if the fluid was not being sheared enough in the piping and was reverting back to its non-sheared state, which had a viscosity value in the thousands.

One theory hypothesized that the pipe and fittings losses were not modeled correctly for handling a non-settling slurry operating in the laminar regime. This theory had a lot of support, since there were still many signs that an inadequate NPSH margin was the issue. First, the pumps were moving fluid satisfactorily and then, within a small reduction in suction tank level, the flow would drop to zero. This sudden performance drop-off fit well with that of a typical “knee curve” for NPSH. Figure 6 shows a typical “knee curve,” where there is a dramatic change in pump head once the pump typically gets past the 3% head drop due to a lack of NPSH margin. Second, on the three pumping systems that had an issue, there were numerous branch tees, valves, and elbows in the suction piping. The two pumping systems that did not have an issue had fewer pipe fittings in the suction piping. Thus, the team began to investigate how the hydraulic losses should be handled for pipe fittings.

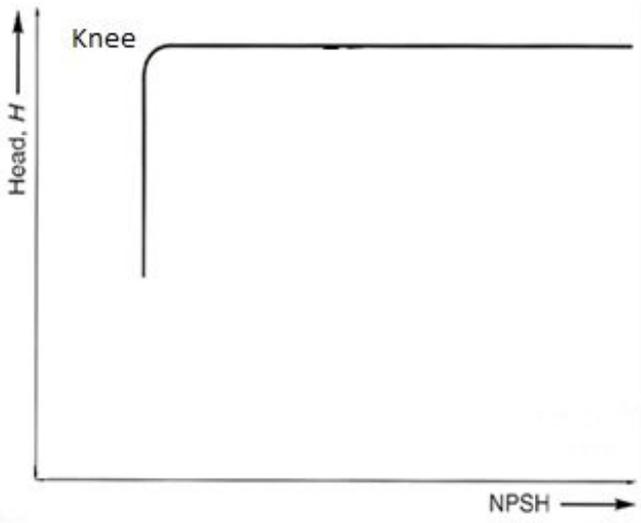


Figure 6. NPSH Breakdown Curve

## CALCULATION OF PRESSURE DROP THROUGH VALVES AND FITTINGS

The calculation of pressure drop in pipe systems has numerous aspects. Two of these are the calculation of frictional pressure drop in pipes, and the calculation of pressure losses in valves and fittings. Included in this list of valves and fittings are:

- Regular valves
- Check valves
- Three-way valves
- Orifices
- Elbows
- Tees
- Area changes
- Screens and filters
- Flow meters

There are two popular methods of calculating pressure drop for valves and fittings: the K factor method and the Equivalent Length method.

The discussion in this paper is limited to incompressible (constant density) applications. Typically, liquid pipe flow behaves in an incompressible manner.

Often valves and fittings are referred to as “minor losses.” This term is synonymous with “valves and fittings.” But it is unfortunate because in some cases the “minor losses” are not minor at all and are in fact major. This paper is an example of such a case.

### Frictional Pressure Drop in Pipes

It is helpful to begin the discussion of pressure drop in valves and fittings by first discussing Newtonian frictional pressure drop in pipes. There are many methods used to calculate pipe pressure drop. The most popular method is based on the Darcy-Weisbach friction factor commonly obtained

through Moody diagrams. The relationship is given by:

$$\Delta P_{pipe} = f \frac{L}{D} \left( \frac{1}{2} \rho V^2 \right) \quad (1)$$

### K Factor Method for Pressure Drop in Valves and Fittings

The K factor method of calculating pressure drop in valves and fittings is given by:

$$\Delta P_{fitting} = K \left( \frac{1}{2} \rho V^2 \right) \quad (2)$$

The K factor is a dimensionless parameter treated as a constant. Hence in its normal form, it has no dependence on Reynolds number.

Inspection of Equations 1 and 2 show a relationship between friction factor and K factor:

$$K \sim f \frac{L}{D} \quad (3)$$

As long as the pipe diameter is constant, the velocity is constant. This means that the Reynolds number is also constant along a pipe. This further means that the K factors of all valves and fittings in a pipe run can be summed together and the overall pressure drop for a horizontal pipe is given by combining Equations 1 and 2:

$$\Delta P_{total} = \left( f \frac{L}{D} + \sum K \right) \left( \frac{1}{2} \rho V^2 \right) \quad (4)$$

### Equivalent Length Method for Pressure Drop in Valves and Fittings

The Equivalent Length method of calculating pressure drop in valves and fittings is based on treating each valve and fitting as an extra length of pipe. The “equivalent length” is just the pressure drop in the valve or fitting equated to the length of pipe required to achieve that same pressure drop. The pressure drop for the fitting is thus similar to Equation 1:

$$\Delta P_{fitting} = f \frac{L_{eq}}{D} \left( \frac{1}{2} \rho V^2 \right) \quad (5)$$

where  $L_{eq}$  is the equivalent length – a value particular to each fitting. In a straight run of pipe, each fitting’s equivalent length can be summed together. Combining Equations 1 and 5 relates the overall pressure drop in a horizontal pipe using a sum of equivalent lengths:

$$\Delta P_{total} = f \frac{(L_{pipe} + \sum L_{eq})}{D} \left( \frac{1}{2} \rho V^2 \right) \quad (6)$$

Table 2 shows representative K factors and their Equivalent Lengths for several types of valves and fittings. Note the Cameron Hydraulic Data source references Crane's Tech Paper 410 (e.g., Crane 1998) for this data.

Also note that Equivalent Length data is given as L/D. Hence the Equivalent Length is obtained by multiplying the L/D value by the diameter of the pipe.

Table 2. Representative Data for K Factors and Equivalent Lengths (Cameron Hydraulic Data, 1995, pp. 3-111 – 3-115, table "Friction Loss in Pipe Fittings")

Valve or Fitting Type	K 1	K 4	K 12	Equivalent Length L/D
	inch (2.5-cm)	inch (10-cm)	inch (30-cm)	
Standard 90° Elbow (threaded)	0.69	0.51	0.39	30
Smooth 90° Elbow (flanged/welded)				
r/D = 2	0.28	0.2	0.16	12
r/D = 10	0.69	0.51	0.39	30
Miter 90° Elbow	1.38	1.02	0.78	60
Valves				
Angle (45° full line)	1.27	0.94	0.72	55
Ball	0.07	0.05	0.04	3
Gate	0.18	0.14	0.1	8
Globe	7.8	5.8	4.4	340
Plug (Straight)	0.41	0.31	0.23	18
Check Lift (V <sub>min</sub> 40)	13.8	10.2	7.8	600
Check Swing (V <sub>min</sub> 35)	2.3	1.7	1.3	100

### Laminar Flow Fitting Calculations

A large majority of industrial pipe systems operate in the high-Reynolds number, turbulent flow regime. Therefore K factors are oriented towards turbulent flow. Traditionally the literature has been ambiguous on how to calculate valve fitting losses in laminar flow conditions. For example, Crane (1988) states on page 2-8:

"The resistance coefficient K is therefore considered as being independent of friction factor and Reynolds number and may be treated as a constant for any given

obstruction (i.e., valve or fitting) in a piping system under all conditions of flow, *including laminar flow.*" (Italics added for emphasis)

and also on page 2-11:

"Equation 2-2 ( $h_L = KV^2/2g$ ) is valid for computing the head loss due to valves and fittings for all conditions of flow, *including laminar flow*, using resistance coefficient K as given in the "K" Factor Table." (Italics added for emphasis)

Note that the referenced "K Factor Table" uses turbulent friction factors ( $f_T$ ) to obtain K. Crane (1988) then refers to examples 4-7 and 4-8 (on pages 4-4 and 4-5) which are titled "Laminar Flow in Valves, Fittings, and Pipe." These two examples apply the K factors under laminar conditions as if they are unchanged from turbulent flow conditions.

Clearly the intent of this paper is not to criticize Crane's 1988 publication, but to discuss some of the literature which has contributed to the current state of ambiguity.

The more recent Crane (2009) reference acknowledges on page 2-10 the effect of laminar flow on K factors but does not offer any direct guidance.

"This results in an increase in the resistance coefficient as the friction factor increases with decreasing Reynolds number in the transition and laminar regions..."

If one takes Equation 3 and applies it to a valve or fitting, then the K factor and equivalent length are related as:

$$K = f \frac{L_{eq}}{D} \quad (7)$$

Since K factor data is more readily available than equivalent length data, it is desirable to use K factor data for laminar as well as turbulent flow. However, it has been realized for many years that K factor data may not be applicable to laminar flow applications. As discussed previously, Crane (1988) discussed standard K factor data as being equally applicable to laminar and turbulent flow. On the other hand, Hooper (1981) argues that standard K factor data is not applicable for laminar flow and a modified "Two-K" method was advocated. Two-K methods use the standard K factor data but also have a second K factor which modifies the Equation 2 pressure drop calculation in a way that is more accurate for laminar flow. The Two-K method calculates pressure drop as:

$$K = \frac{K_1}{Re} + K_\infty \left( 1 + \frac{1}{D_i} \right) \quad (8)$$

where  $K_1$  is the K factor at Reynolds number of 1,  $K_\infty$  is the high Reynolds number K value at large diameters and  $D_i$  is the internal diameter in inches. Values of  $K_1$  were obtained by test and included in Hooper (1981).

To make things even more accurate – at the expense of additional complication – a "Three-K" method has been

advocated by Darby (2001):

$$K = \frac{K_1}{Re} + K_i \left( 1 + \frac{K_d}{D_i^{0.3}} \right) \quad (9)$$

where  $K_1$  means the same as in Equation 8 and  $K_i$  and  $K_d$  combine to determine the turbulent/high Reynolds number K factor. Table 3 shows the additional K factors suggested for use in the Three-K method.

Table 3. K Factors Constants for Two-K and Three-K Methods (Hooper, 1981 and Darby, 2001)

Valve or Fitting Type	$K_1$ Eq. 8, 9	$K_\infty$ Eq. 8	$K_i$ Eq. 9	$K_d$ Eq. 9
Standard 90° Elbow (threaded)	800	0.51	0.14	4.0
Smooth 90° Elbow (flanged/welded )				
r/D =1.5	800	0.2	0.071	4.2
r/D =6	800	N/A	0.075	4.2
Miter 90° Elbow	1000	1.15	0.27	4.0
Valves				
Angle (45° full line)	1000	N/A	0.25	4.0
Ball	500/ 300	0.15	0.017	4.0
Gate	300	0.1	0.037	3.9
Globe	1500	4.0	1.7	3.6
Plug (Straight)	300	N/A	0.084	3.9
Check Lift ( $V_{min}$ 40)	2000	10.0	2.85	3.8
Check Swing ( $V_{min}$ 35)	1500	1.5	0.46	4.0

Besides the additional complexity of Equations 8 and 9 compared to Equation 2, a significant issue is finding data for general valves and fittings in order to use Equations 8 and 9. One can use the data in Table 3, but what if such data is not available?

What is needed is a way to use readily available K factor data in cases of laminar flow, without having to seek out Two-K and Three-K data. The following discussion covers such a method.

An appropriate equivalent length can be determined from standard turbulent condition K factor data by looking at large Reynolds numbers and hence fully turbulent conditions using Equation 7:

$$L_{eq,turb} = \frac{K_{turb} D}{f_{turb}} \quad (10)$$

Assuming for the moment that the equivalent length for laminar flow is the same as for turbulent flow, then Equation 10 can be used for laminar conditions:

$$L_{eq,lam} = \frac{K_{lam} D}{f_{lam}} \quad (11)$$

Equating equations 10 and 11 and generalizing the K factor for any turbulent or laminar condition obtains:

$$K = K_{turb} \frac{f}{f_{turb}} \quad (12)$$

Equation 12 will be called the Adjusted Turbulent K Factor (ATKF) method. To apply the method, take the standard (turbulent) K factor and multiply it by the relevant (upstream) pipe friction factor,  $f$ , at actual conditions (and Reynolds number) and divide it by the turbulent friction factor,  $f_{turb}$ , evaluated at a very large Reynolds number ( $10^8$ ) for the upstream pipe.

### COMPARISON OF ATKF METHOD IN EQUATION 12 TO PUBLISHED METHODS

A study was undertaken to evaluate Equation 12 against published data. A comparison for a 10 foot (3 meter) long, steel pipe with fluid with the density of water but various viscosity values (to adjust Reynolds number) was performed. Two cases were considered: 2-inch (5-cm) and 24-inch (60-cm) diameter. Inside each pipe were 12 elbows of r/D = 1.5.

#### Five cases compared

1. No fittings in the pipe
2. Fittings of 12 elbows of 1.5 r/D (with a total K of 3.17 for 2-inch diameter and 2.02 for 24-inch), but no corrections. This assumes the elbow laminar K factor is the same as the turbulent K factor.
3. Darby 3-K method for 12 elbows (see Darby, 2001, pp. 209)
4. Equivalent length of 12 elbows with  $L_{eq}/D = 16$  (see Darby, 2001, pp. 209 and Cameron Hydraulic Data, 1995)
5. Adjusted K factor method, Eq. 12

Figure 7 shows the results for dimensionless head gradient (loss of head per length of pipe) on a log-log plot for the 2-inch (5-cm) case. One would not expect Case 1 to match the others, as no fittings were included. It is a baseline case included for comparison. Hence the head gradient was less than the other cases. However Case 1 does illustrate how Case 2 works in the laminar regime. It is apparent that Case 2 (with the uncorrected, standard turbulent K factor) grossly underpredicts the head loss at low Reynolds numbers. Case 2 agrees with Cases 3-5 at high Reynolds number, but agrees with Case 1 at low Reynolds number. This is a result of using turbulent, high Reynolds number K factors for low Reynolds number applications.

Cases 3, 4 and 5 all agree fairly well throughout the entire range of Reynolds numbers.

Figure 8 shows the results on a log-log plot for 24-inch (60-cm) pipe. The trends are the same as in Figure 7, except there is a larger difference at Reynolds numbers of 10,000-

100,000.

The conclusion from Figures 7 and 8 is that the ATKF method from Equation 12 allows reasonable adjustment of turbulent K factor pressure loss data for valves and fittings to laminar applications.

Also note that Equation 11 was based on the assumption that the Equivalent Length method (Case 4) applies equally to laminar and turbulent flow. Inspection of Figures 7 and 8 for Cases 3 and 4 shows that use of the Equivalent Length method applied to laminar flow gives comparable results to the Three-K method (Case 3). Thus, this assumption appears to be quite reasonable.

Because Figures 7 and 8 are log-log plots, the magnitude of the differences between Cases 3-5 appear to be smaller than they are in an absolute sense. Obviously Case 5 (Equation 12) is closer to Cases 3 and 4, while Case 2 (which ignores laminar adjustments) is different by 1-2 orders of magnitude at low Reynolds numbers. However, at Reynolds numbers equal to 1, there is a difference of 25-40% between Cases 3, 4 and 5. Arguably, Case 3 (the Three-K method) is the most accurate of all the methods. The ATKF method of Case 5 (Equation 12) underpredicts the pressure drop by about 25% in Figure 8 for 24-inch (60-cm) pipe. Tables 4 and 5 show the actual K factors behind Figures 7 and 8 so the various methods can be more precisely compared.

It is therefore concluded that the ATKF method of Equation 12 is a reasonable substitute for the Three-K method over a wide range of Reynolds numbers, and is recommended for use should laminar data be unavailable. The Case 5 ATKF method is much more accurate than Case 2 at low Reynolds numbers.

Moreover, the ATKF method of Equation 12 is suitable for all valve and fitting calculations whether they be laminar, turbulent, Newtonian or non-Newtonian. In cases where turbulent flow exists, the adjustment factor will just converge to "1" so no impact will be observed on the conventional calculation methods. In cases where more precise laminar data is available for valves and fittings then such data would be preferred over the ATKF method.

Note that the ATKF method agrees better with the Three-K method than does the Equivalent Length method at low Reynolds numbers. Even so, use of the Equivalent Length method in its normal form (Case 4) rather than the standard, unadjusted K factor method (Case 2) would have yielded more reasonable results in this case study. The entire pumping issue might have been avoided. However, similar to the Two-K and Three-K methods, Equivalent Length data is not as readily available as K factor data and its use in this case study might not have been practical.

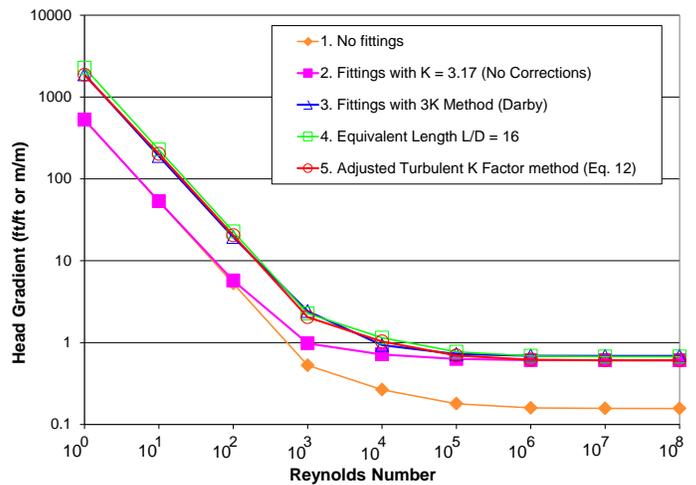


Figure 7. Example for 2-inch (5-cm) pipe of dimensionless head gradient for various calculation methods shows Adjusted Turbulent K Factor (Equation 12, Case 5) closely follows the 3-K method (Case 3) and Equivalent Length method (Case 4).

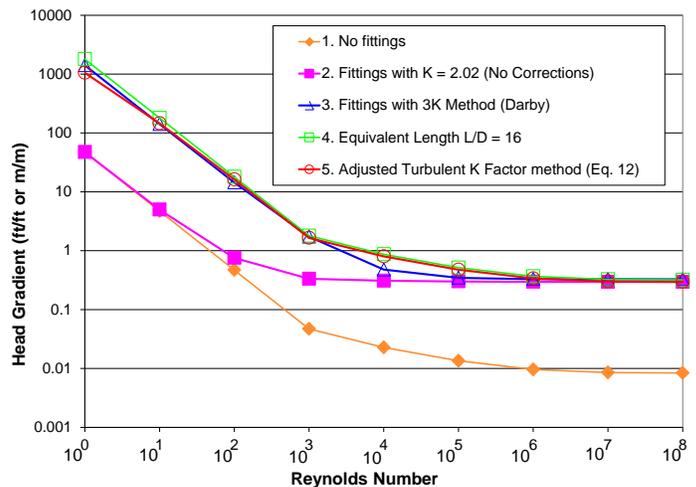


Figure 8. Example for 24-inch (60-cm) pipe of dimensionless head gradient for various calculation methods shows Adjusted Turbulent K Factor (Equation 12, Case 5) closely follows the 3-K method (Case 3) and Equivalent Length method (Case 4).

Table 4. K factors representing 12 elbows vs. Reynolds numbers for Cases 2-5 with 2-inch (5-cm) pipe

Case	2	3	4	5
Reynolds Number	K constant	3K (Darby)	Equiv. Length	ATKF Eq. 12
1	3.2	9,585	12,268	9,367
10	3.2	962	1,227	1,049
100	3.2	99.5	123	107
1,000	3.2	13.3	12.3	10.7
10,000	3.2	4.7	6.2	5.4
100,000	3.2	3.8	4.2	3.6
1,000,000	3.2	3.7	3.7	3.2
10,000,000	3.2	3.7	3.7	3.2
100,000,000	3.2	3.7	3.6	3.2

Table 5. K factors representing 12 elbows vs. Reynolds numbers for Cases 2-5 with 24-inch (60-cm) pipe

Case	2	3	4	5
Reynolds Number	K constant	3K (Darby)	Equiv. Length	ATKF Eq. 12
1	2.0	9,593	12,272	7,063
10	2.0	961	1,227	988
100	2.0	98.2	123	111
1,000	2.0	11.8	12.3	11.3
10,000	2.0	3.2	6.0	5.5
100,000	2.0	2.3	3.5	3.3
1,000,000	2.0	2.3	2.5	2.3
10,000,000	2.0	2.3	2.2	2.1
100,000,000	2.0	2.2	2.2	2.0

## APPLICATION OF ADJUSTED TURBULENT K FACTOR METHOD TO NON-NEWTONIAN FLUIDS

The previous discussion of the ATKF method (Equation 12) was focused on laminar flow. The applications of interest which had the operational problems were not only laminar flow, but *non-Newtonian* laminar flow. However, it appears that the ATKF method (Equation 12) works equally well, as long as the proper non-Newtonian friction factor,  $f$ , is used in Equation 12.

The slurry fluid involved in the operational problems was modeled using a power law model. This model uses raw rheological data for shear stress and shear rate to obtain power law constants  $K_{pl}$  and  $n$ .  $K_{pl}$  has units of viscosity and  $n$  is dimensionless. These constants relate fluid shear stress to shear

rate as:

$$\tau = K_{pl} \dot{\gamma}^n \quad (13)$$

The laminar relationship is given by Darby (2001, p. 165 – converted here from a Fanning to a Moody friction factor):

$$f_{pl,lam} = \frac{64}{Re_{pl}} \quad (14)$$

where:

$$Re_{pl} = \frac{8D^n V^{(2-n)} \rho}{K_{pl} (2(3n+1)/n)^n} \quad (15)$$

Many power law fluid applications operate in the laminar regime, and hence Equation 14 is often the only equation needed for friction factor calculation. The complicated part of using Equation 14 is the original determination of  $K_{pl}$  and  $n$  from Equation 13.

A combined laminar and turbulent friction relationship factor for a power law fluid is given by Darby (2001, pp. 166-167) and is not discussed here for brevity, since the interest in this project was laminar flow.

Considering Equation 14, the important issue to grasp is that when using the ATKF method of Equation 12 for non-Newtonian fluids, it is not merely a laminar friction factor that is used. Rather, a laminar, *non-Newtonian* friction factor as calculated by Equation 14 is used in Equation 12. This then allows an adjustment of standard, turbulent K factors for use in laminar, non-Newtonian applications.

Fortunately, the ATKF method had been previously implemented in the commercial software used in this case study as an add-on module (Applied Flow Technology, 2010). The team had not been aware of this module and after consultation with the software developer expanded the pumping system model to apply the ATKF method.

## ANALYSIS OF CASE HISTORY PUMPING SYSTEMS

The pumping system model calculations were then modified to incorporate the ATKF Method (Equation 12). Since the goal was to empty the vessels, the calculations for NPSH margin were done at zero level. Table 6 shows the values for NPSH at zero tank level with the implementation of the ATKF Method.

Table 6. System Hydraulic Predictions Using Non-Newtonian Pipe Friction with ATKF Method Before Modifications to Piping

#	Flow gpm (m <sup>3</sup> /hr)	$\Delta P$ Suction Pipe psid (kPa)	$\Delta P$ Suction Pipe Fittings psid (kPa)	<sup>1</sup> NPSH <sub>A</sub> ft (m)	NPSH <sub>R</sub> ft (m)	NPSH Margin ft (m)
1	107 (24.3)	0.6 (4.1)	12.3 (84.8)	-1.3 (-0.4)	3.0 (0.9)	-4.3 (-1.3)
2	49 (11.1)	0.2 (1.4)	16.0 (110.3)	-3.0 (-0.9)	2.2 (0.7)	-5.2 (-1.6)
3	95 (21.6)	0.2 (1.4)	7.6 (52.4)	10.0 (3.0)	2.7 (0.8)	7.3 (2.2)
4	60 (13.6)	1.2 (8.3)	11.5 (79.3)	-0.2 (-0.1)	2.3 (0.7)	-2.5 (-0.8)
5	74 (16.8)	0.1 (0.7)	0.4 (2.8)	11.2 (3.4)	2.4 (0.7)	8.8 (2.7)

<sup>1</sup>Although having a NPSH<sub>A</sub> less than zero is physically impossible, the authors have chosen to show the full impact of the hydraulic losses to give the reader a better understanding of the scale of the issue.

Table 6 shows that NPSH would only become an issue on pumping systems #1, #2, and #4, which correlates exactly with what was experienced in the field. The predictions in Table 1 and 6 differ only because of the use of the ATKF Method in Table 6. Pumping systems #3 and #5 never had an issue with leaving a heel in the suction vessel, which also correlates with the results using the ATKF Method. At this point, the team felt strongly that they had found the reason for the heels left in the suction vessels.

The increased frictional loss predicted by the ATKF Method also explains why pumping system #2 did not deliver the needed pressure at the user. All of the pipe fittings in the discharge line were causing significant pressure drop. In addition, the ATKF Method explains that pumping systems #1, #4, and #5 had inadequate flow rates due to higher than anticipated pressure drop in the discharge piping.

## CORRECTIVE ACTIONS TAKEN

### NPSH Margin Corrections

In order to correct the NPSH margin issue on pumping systems #1, #2, and #4, the suction piping needed to be changed. The details of the changes are discussed here:

- A first priority was given to eliminating branch flow tees in the system entirely and, if a change in direction of the flow was needed, a large radius elbow with an  $r/D = 3$  was used.
- The number of changes in the piping direction was minimized to eliminate elbows and tees.

- Anywhere there was a change in piping diameter, a conical shaped transition was used with a taper angle of 15 degrees, if possible. It was important to eliminate abrupt changes in piping sizes to minimize losses.
- Changes in pipe sizes were made to increase the pipe size at least one size larger than the pump suction. Before this change, a couple of the systems had suction piping runs that were around 40 ft (12.2 m) long and the piping for the whole run was the same size as the pump suction. On system #4, there were more losses in the suction piping itself that had to be corrected to improve the NPSH<sub>A</sub>.

The results of the suction piping changes using the ATKF Method for both before and after calculations are seen in Table 7.

### Pressure and Flow Inadequacy Corrections

Pumping #1 and #2 had inadequate flow and pressure delivery. These systems had numerous branch flow tees in the discharge piping as well as other fittings. The same strategy was used in the discharge piping as in the suction piping to eliminate or minimize friction losses while analyzing the system with the implementation of the ATKF Method. In addition, it was predicted that it would be necessary to increase the impeller diameter in systems #1, #2, and #4.

Table 7. System Hydraulic Predictions Using Non-Newtonian Pipe Friction with ATKF Method After Modifications to Piping and Pump Impeller Diameter Increase

#	Flow gpm (m <sup>3</sup> /hr)	$\Delta P$ Suction Pipe psid (kPa)	$\Delta P$ Suction Pipe Fittings psid (kPa)	NPSH <sub>A</sub> ft (m)	NPSH <sub>R</sub> ft (m)	NPSH Margin ft (m)
1	135 (30.7)	0.5 (3.4)	7.1 (49.0)	6.3 (1.9)	3.4 (1.0)	2.9 (0.9)
2	95 (21.6)	0.2 (1.4)	11.3 (77.9)	7.0 (2.1)	2.7 (0.8)	4.3 (1.3)
3 <sup>2</sup>	95 (21.6)	0.2 (1.4)	7.6 (52.4)	10.0 (3.0)	2.7 (0.8)	7.3 (2.2)
4	78 (17.7)	0.5 (3.4)	6.8 (46.9)	4.3 (1.3)	2.4 (0.7)	1.9 (0.6)
5 <sup>2</sup>	74 (16.8)	0.1 (0.7)	0.4 (2.8)	11.2 (3.4)	2.4 (0.7)	8.8 (2.7)

<sup>2</sup>No changes were made to this system.

## RESULTS

After making the piping changes and impeller alterations, the results were dramatic:

- All suction vessels could be pumped down to nearly empty levels.
- System flows slightly exceeded calculated values. Pinch valves were closed in the discharge line to regulate flow.
- The pressure delivered to the user on system #2 exceeded predicted values. Pinch valves were closed in the discharge line to regulate pressure and eventually the impeller was trimmed to reduce the pressure being delivered to the end user. Pumping systems #1 and #4 also delivered higher flows than expected, but this was acceptable, since it increased the speed of the batch. The additional flows and pressure were unexpected, and it was assumed the liquid was not going back to its originally pre-sheared state after going through the pump. The Metzner-Otto rule suggests that a centrifugal pump would apply a shear rate of approximately 11 times the rpm of the pump. It is possible this much shear caused the fluid to stay in a heavily sheared state for quite some time, even after the liquid entered regions of lower shear. Thus, the viscosity was much lower in the discharge line than predicted and thus the hydraulic losses were lower.

The alterations were considered a major success, and it allowed the plant to go into full production. The increased flow and pressure values were also reflected in the previous products made by the plant, and production levels were able to be increased for those products as well.

## CONCLUSIONS AND RECOMMENDATIONS

- For a non-settling slurry operating in or near its laminar regime, the method proposed in this paper for estimating the pressure drop through fittings provided excellent results in the case history described.
- The NPSH available can be significantly impacted by pipe fittings in the suction line. Even though some of these fittings may be using for clean out (e.g., branch flow tees), their use should be balanced against providing adequate NPSH for the pump.
- Vessel nozzle outlet shapes are critical to the preservation of NPSH margins in non-settling slurry pumping systems operating in or near the laminar regime. Sharp edge outlets should be avoided if possible, and the use of rounded edge outlets should be employed.
- As a guideline, the  $NPSH_A$  should exceed the  $NPSH_R$  by a minimum of 5 ft (1.5 m), or be equal to 1.35 times the  $NPSH_R$ , whichever is greater. For example, for an  $NPSH_R$  of 10 ft (3.0 m), the  $NPSH_A$  should be a minimum of 15 ft (4.5 m).
- Downstream fittings can also cause excessive pressure drop and flow reduction. It is important to minimize fittings where possible in the piping downstream of the pump.
- The shear added by a centrifugal pump to the liquid is significant. It is estimated per the Metzner-Otto rule that a centrifugal pump shears the fluid at 11 times the rpm of the

pump. Some fluids do not quickly revert back to their pre-sheared state after undergoing such a high level of shear, and this may cause pressure drop calculations through fittings to be overly conservative downstream of the pump.

- Care should be taken when using hydraulic analysis software to ensure that the fittings are being analyzed properly from a pressure drop standpoint.

## NOMENCLATURE

$D$	= Diameter of pipe
$D_i$	= Inner Diameter of pipe
$f$	= friction factor
$f_{lam}$	= laminar friction factor
$f_{pl,lam}$	= friction factor for power law under laminar flow conditions
$f_{turb}$	= turbulent friction factor
$L, L_{pipe}$	= Length of pipe
$L_{eq}$	= Equivalent length of pipe due to fittings
$L_{eq,lam}$	= Equivalent length of pipe due to fittings under laminar flow conditions
$L_{eq,turb}$	= Equivalent length of pipe due to fittings under turbulent flow conditions
$K$	= K factor of valve or fitting
$K_d$	= Parameter in Three-K method of Equation 9
$K_i$	= Parameter in Three-K method of Equation 9
$K_{lam}$	= K factor of valve or fitting under laminar flow conditions
$K_{pl}$	= Constant for power law in Equation 13
$K_{turb}$	= K factor of valve or fitting under turbulent flow conditions
$K_1$	= Parameter in Two-K method of Equation 8
$K_\infty$	= Parameter in Two-K method of Equation 8
$n$	= Constant for power law in Equation 13
NPSH	= Net positive suction head (m or ft of liquid)
$NPSH_A$	= Net positive suction head available (m or ft of liquid)
$NPSH_R$	= Net positive suction head required (m or ft of liquid)
Re	= Reynolds number
$Re_{pl}$	= Reynolds number for power law, Equation 15
$V$	= velocity of liquid
$\rho$	= density of liquid
$\dot{\gamma}$	= shear rate
$\tau$	= shear stress

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