

# A Proposed Guideline For Applying Waterhammer Predictions Under Transient Cavitation Conditions

## Part 2: Imbalanced Forces

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## **A PROPOSED GUIDELINE FOR APPLYING WATERHAMMER PREDICTIONS UNDER TRANSIENT CAVITATION CONDITIONS PART 2: IMBALANCED FORCES**

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### **ABSTRACT**

Waterhammer analysis (herein referred to as Hydraulic Transient Analysis or simply “HTA”) becomes more complicated when transient cavitation occurs (also known as liquid column separation). This complication is exacerbated when trying to predict imbalanced forces as this often involves comparing pressure times area (“PxA”) forces at two locations (for example at elbow pairs). Whereas the pressure at each elbow location has increased uncertainty because of transient cavitation, the difference in PxA forces at elbow pairs involves subtracting one potentially uncertain pressure from another uncertain pressure. Exacerbating this uncertainty yet further, the existence of vapor in a liquid system can dramatically affect the fluid wavespeed and, hence, the timing of the pressure wave travel between two locations such as elbow pairs; so the pressure calculated at each location would not actually occur at exactly the same time.

This Part 2 discusses methods of accounting for uncertainty in HTA imbalanced force predictions due to cavitation. The criteria in this paper assume that cavitation in the HTA has been assessed and accepted per the criteria in Part 1 of this paper.

A guideline is proposed for accepting and applying such results and, in particular, makes recommendations on safety factors to use in pipe stress analysis for different cases. The specific recommendations depend on numerous factors including:

- Presence or absence of cavitation in hydraulically connected or isolated parts of the system
- If cavitation occurs, whether the peak forces occur before or after cavitation first occurs

- Size of the cavitation vapor volumes with respect to the computing volumes
- Use of point forces as a conservative substitute in place of potentially less certain elbow pair forces or the manual assessment of maximum envelope values for the force.

Situations are discussed where waterhammer abatement is recommended to reduce hydraulic transient forces, and for increasing confidence in HTA results in specific cases. The result is a proposed comprehensive and pragmatic guideline which practicing engineers can use to perform waterhammer analysis and apply imbalanced force predictions to pipe stress analysis.

### **KEYWORDS**

water hammer, fluid transient, hydraulic transient analysis (HTA) transient cavitation, liquid column separation, ASME B31.3, Discrete Vapor Cavity Model (DVCM), Discrete Gas Cavity Model (DGCM), transient force, imbalanced force, dynamic analysis, dynamic load factor (DLF)

### **OVERVIEW**

A byproduct of HTA is the generation of essential parameters to predict imbalanced forces due to waterhammer. The occurrence of transient cavitation (frequently called liquid column separation) greatly complicates the task of determining maximum forces. If one wants to calculate imbalanced forces from waterhammer with increased confidence, it is best to avoid transient cavitation altogether. However, that is not always possible. As a result, increased safety factors are recommended as discussed in this guideline.

As discussed in Part 1 of this paper (Stewart, Walters, Wunderlich and Onat [1]), the authors collaborated on a project involving radioactive fluid transport. The collaboration involved bringing together HTA Engineers and Pipe Stress Engineers to create criteria for accepting HTA results and imbalanced force predictions. The HTA Engineers documented these criteria in a software validation report. The collaboration was unique in that it involved not only the engineering design firm but also the developer of the HTA software.

The authors are not aware of any previously published guidelines for accepting predictions of imbalanced forces from waterhammer. The engineering design firm from the first and third author needed pragmatic guidance on how to interpret and apply HTA pressure and imbalanced force predictions.

The traditional approach for predicting imbalanced forces at, for example, elbow pairs, uses differences in pressure multiplied by area at the elbows. A more complete force balance includes friction and fluid momentum which can result in substantially different predictions. See Wilcox and Walters [2]. Whichever method is used, the presence of transient cavitation is a complicating factor.

As in Part 1, considerable judgment was required in assembling this proposed guideline. In some ways, even more judgment was needed for this Part 2 on imbalanced forces. The authors' driving purpose was to create something pragmatic and actionable. In an area as uncertain as HTA with transient cavitation, this required some difficult decisions on ambiguous issues. As a result, there is ample room for debate and disagreement which the authors welcome. Engineers wishing to use these criteria should consider following the requirements of ASME B31.3 Para 300 (c)(3) [3].

In order to use this proposed guideline, the HTA Engineer needs to be able to calculate maximum forces as well as force-time histories. The HTA software used, *AFT Impulse*, is commercially available and discussed in Applied Flow Technology [4] and Ghidaoui et al. [5]. It includes features required for a traditional or complete force balance (Wilcox and Walters [2]) and generates both maximum force output and force-time histories. Part 1 (Stewart et al. [1]) provides more information on HTA software solution methodology and cavitation models (DVCM and DGCM). Other HTA software can be used with this guideline as long as it can export transient output for independent pipe force calculation (e.g., in Excel).

The guideline proposed in this paper applies most directly to MOC-based (Method of Characteristics) waterhammer software and designs which apply ASME piping code. The authors have made their best effort to generalize the internal document we developed into this guideline to make it as widely applicable as possible. The structure we have created here should prove useful and adaptable to situations which do not use MOC software or ASME piping code.

The guideline consists of the following sections 1.0-5.0. Later in this paper, after concluding the proposed guideline, the authors discuss some of the reasoning behind the decisions they made.

## ABBREVIATIONS

CAC	Cavitation Acceptance Criteria
CSF	Cavitation Safety Factor
CSM	Cavitation Safety Margin
CVR	Cavitation Volume Ratio
CVR <sub>MAX</sub>	The maximum of all CVR values for all pipe computing segments over all time steps of the simulation [1]
DGCM	Discrete Gas Cavity Model – a model used in waterhammer analysis to simulate the formation and collapse of vapor cavities
DLF	Dynamic Load Factor
DVCM	Discrete Vapor Cavity Model – a model used in waterhammer analysis to simulate the formation and collapse of vapor cavities
HTA	Hydraulic Transient Analysis (waterhammer analysis)
HTF	Hydraulic Transient Force (imbalanced force on piping that occurs as a result of waterhammer)
MOC	Method of Characteristics
SF	Safety Factor
SM	Safety Margin
SME	Subject Matter Expert

## 1.0 GUIDELINE INTRODUCTION

This criterion follows best practices provided by the authors and requires the use of judgment by the user. In some cases, deviation from the criteria may be warranted; in these cases, HTA Engineers should consult with their Subject Matter Expert (SME) and document why the deviation is necessary, and the alternate means used for acceptance. In the end, the results should seem reasonable per the judgment of the HTA Engineer.

Before applying this criterion, the presence and extent of transient cavitation (hereafter referred to more simply as “cavitation”) must be verified by separate criteria (Stewart, Walters, Wunderlich and Onat [1]). The presence of cavitation during a transient event affects wavespeed and timing as it moves through a specific location. Hydraulic Transient Forces (HTFs) frequently rely on the comparison of pressures in two distinct locations at a particular time. Inaccuracies in wavespeed and timing due to the presence of cavitation can result in inaccurate HTFs when using force pairs (i.e., elbow pairs), even when the pressure magnitude is considered reliable.

## 2.0 DEFINITIONS

**Animation** – A feature in some HTA software whereby a parameter such as pressure is plotted vs. length along a contiguous pipe path and animated over time (e.g., Applied Flow Technology [4]).

**Cavitation Acceptance Criteria (CAC)** – Criteria for accepting HTA pressure predictions resulting from transient cavitation. CAC includes recommended safety factors. These are documented in Part 1, Stewart et al. [1]. Note that accepting HTA

pressure predictions is not the same as accepting HTA imbalanced force predictions.

**Dynamic Load Factor (DLF)** – A dynamic load factor represents the maximum theoretical effect of momentum on a dynamic structural analysis. Applying a DLF is a simple, conservative method accounting for momentum in a static structural analysis. Maximum loads output from HTA software is multiplied by a DLF before being used in a static pipe stress analysis. Time history loads used in a dynamic piping or structural analysis does not require a DLF.

**Force Pairs** – A method used to calculate hydraulic transient forces based on differential pressures and momentum at two specific locations (Wilcox and Walters [2]). A force pair represents the two ends of a piping segment that would usually cancel out each other's static pressure force. The user selects the location of the nearest upstream and downstream bend or elbow. HTA software then determines the maximum pressure differential that occurs between these two locations.

**Minimum Pressure** – When performing a manual assessment of maximum envelope forces, the minimum pressure used depends on the presence of cavitation. Where force pair results are being supplemented by a manual check of forces in surrounding pipe, the maximum envelope force may be based on the pressure at the base of the transient wave under consideration. For example, if transient waves and operating states never drop below 50 psi (345 kPa), this may be used as the minimum pressure. Where cavitation is present, the pressure at the base of the transient wave may not be reliable, so the minimum pressure must be selected more carefully when calculating the maximum envelope force. The most conservative option is to use the vapor pressure of the fluid as the minimum pressure.

**Numerical Model Noise** – Pressure spikes that only last for one or a few time steps and/or are very sensitive to small changes in model input parameters that are a mathematical anomaly of the calculation (i.e., artifact of the cavitation model). These do not represent conditions that would occur in the real world.

**Point Force** – A method used to calculate hydraulic transient forces based on pressure and momentum in the pipe compared to a user-defined reference pressure. The minimum pressure as defined above may be used as the reference pressure. Point forces can calculate unbalanced forces such as at hose connections and atmospheric discharges. Point forces are not affected by wave timing errors because they only rely on the transient pressure at one point. They can also provide a very conservative but reliable envelope of maximum force in a pair of elbows that is not affected by wave timing errors; unfortunately, this assessment is often overly conservative, especially for situations where the transient pressure is always less than the system steady-state operating pressure.

**Safety Factor** – A factor applied to forces generated by the HTA software to account for the uncertainties in the calculation results. Where cavitation is present, larger safety factors are recommended, and where cavitation results are used to generate force pairs, the safety factors are further increased to address wavespeed and timing errors.

**SME** – Subject Matter Expert (e.g., the Responsible Hydraulic Transient Engineer or a consultant).

### 3.0 SAFETY FACTOR / SAFETY MARGIN

**Very Large** – 200% Safety Margin or 3:1 Safety Factor against rupture

**Large** – 100% Safety Margin or 2:1 Safety Factor against rupture

**Moderate** – 50% Safety Margin or 1.5:1 Safety Factor against rupture

**Small** – 25% Safety Margin or 1.25:1 Safety Factor against rupture

Henceforth in this document, to distinguish between normal language usage of the preceding words and the usage in the present context of safety factor/safety margin magnitude, the words **Very Large**, **Large**, **Moderate** and **Small** are bolded and capitalized when used in the present context.

Due to the uncertainties associated with forces calculated by HTA software, the HTA Engineer must always multiply HTFs by the safety factor specified in these criteria. For a 200% Cavitation Safety Margin (CSM) or 3:1 Cavitation Safety Factor (CSF), multiply calculated forces by 3; for a 100% CSM or 2:1 CSF, multiply calculated forces by 2; for a 50% CSM or 1.5:1 CSF, multiply the calculated forces by 1.5; for a 25% CSM or 1.25:1 CSF, multiply the calculated forces by 1.25. This force may later be doubled again by the Pipe Stress Engineer to provide a DLF. Where piping is rigid, and DLF are considered conservative, the application of multiple factors could be considered excessive and be moderated as determined by the SME.

HTA Engineers and Pipe Stress Engineers must communicate HTFs with clearly communicated load factors in a consistent format. Whether or not forces being discussed already include a DLF is commonly a source of miscommunication in stress analysis. Adding in a CSF to the discussion further compounds the potential for confusion since CSF and DLF may have the same numerical value and HTFs sometimes need to be multiplied by a DLF, CSF, both or neither. See the examples below for best practice communication:

- When providing a maximum HTF from HTA software tabular output:  
“The maximum HTF is 200 lb<sub>f</sub> (890 N) based on HTA software output of 100 lb<sub>f</sub> (445 N) and a 2:1 Cavitation Safety Factor; this does not include a DLF.

This force is not for use in a dynamic pipes stress analysis.”

- When describing a force-time history plot:  
“This force time history must be multiplied by the Cavitation Safety Factor of 2:1. This force time history does not include a DLF. The maximum HTF is 200 lbf (890 N) based on the peak force of 100 lbf (445 N) and a 2:1 Cavitation Safety Factor; this does not include a DLF.”
- When describing a manually assessed HTF:  
“The maximum HTF is manually assessed as 166 lbf (738 N) based on a maximum 33 psi (228 kPa) wavefront multiplied by a 3.36 in<sup>2</sup> (21.7 cm<sup>2</sup>) pipe area and a 1.5:1 Cavitation Safety Factor; this does not include a DLF. This force is not for use in a dynamic pipes stress analysis.”

#### 4.0 GENERAL GUIDELINES

At each location where forces are to be calculated, the user enters the location of three force endpoints. The elbow of interest is one location, the nearest upstream elbow is another, and the nearest downstream elbow is another.

The criteria used for selection of safety factors depend on the level of cavitation (**None**, **Limited**, **Major**, or **Extreme** – see Definitions in Stewart et al. [1], Part 1). The determination of the cavitation level shall be based on a scenario run with the least number of sections that provide less than 15% wavespeed roundoff error. The safety factor derived from the level of cavitation is then applied to HTFs that are calculated using an optimized level of sectioning ( $\leq 10\%$  wavespeed roundoff error) – see Stewart et al. [1] section 2.0 for more on this topic. Refer to the “Limitations Due to Sectioning” in section 4.2 in this Part 2 for a discussion regarding the impact of MOC sectioning on force calculation.

Using the force pair method to calculate HTFs with cavitation introduces uncertainty and risk of errors. Where force pair data calculated in the presence of cavitation is used, the best practice is to increase the safety factor to account for this uncertainty. The more cavitation present, the greater the uncertainty and the larger the safety factor needs to be. The safety factor should start with the same value used for pressure results in the CAC (Part 1, Stewart et al. [1]) and be increased to the next highest level defined in the cavitation criteria to account for wavespeed and timing error. For example, if the Part 1 CAC indicates that a 1.5:1 CSF (**Moderate**) is required in determining the maximum pressures, then a 2:1 CSF (**Large**) should be used for determining the HTFs.

Pressure spikes due to numerical model noise may be discarded from manual or automatically generated force outputs by the user.

#### 4.1 Use of Cavitation Models When Determining Forces

The Discrete Vapor Cavity Model (DVCM) and the Discrete Gas Cavity Model (DGCM) comprise two mathematically different cavitation models (Stewart et al. [1] and Applied Flow Technology [4]). The DGCM is generally more accurate for

matching wavespeed and timing effects from cavitation [1] and Liou [6], so where the two models provide different HTFs, greater accuracy is expected from the DGCM.

#### 4.2 Limitations Due to Sectioning

Selection of force locations in MOC software is limited by the number of available pipeline computing stations. MOC software divides pipe segments into sections of uniform length delineated by computing stations and calculates pressures at each of these stations. A typical approach in MOC software is to round desired force locations entered by the user to the nearest existing computing station. Thus, calculated forces are typically not at the exact location desired by the user.

Increasing the number of sections allows finer control over the location of the forces. Adding additional stations comes at the cost of substantially increasing the calculation time and can increase the potential for numerical model noise in the calculation results. Calculation run time is typically quadrupled with each doubling of the number of stations in MOC based software (Applied Flow Technology [4]). To avoid unnecessarily long run times, the number of sections is chosen to balance run time and calculation accuracy.

Ideally, force pairs are provided for every elbow, tee and direction change in the piping system being considered. This level of detail may require extensive effort that may not always be warranted. Additionally, HTFs calculated during preliminary stages of design and based on preliminary piping layout may be thought of as general representations of expected final HTFs. In these cases, force pair locations should be selected carefully, and extra precautions are required. Representative force pairs should be located where HTFs are expected to be the highest such as segments of pipe with the longest expected distance between elbows and where the piping is expected to be the most vulnerable to HTFs – such as locations with the most flexibility and the largest stress intensification factors (ASME [3]). It is possible for constructive interference of pressure waves to briefly create a pressure spike in a specific location that is significantly higher than the maximum pressure in adjacent locations. If such a pressure spike occurs at an elbow, the HTF from adjacent locations does not represent a conservative estimate for the elbow in question.

Both fluid momentum and pressure contribute to the forces on an elbow in a piping system. Hydraulic transients typically create unsteady flow and pressure resulting in elbows with different reaction forces. At typical velocities and pressures seen in industrial piping systems the force due to unbalanced pressure or PxA dominates over forces caused by momentum. This procedure specifically addresses differential pressure forces; however the same methods may be applied to differential momentum forces. For example consider an elbow in a piping system with 2.067 inch (5.25 cm) inside diameter, operating at 100 psi (690 kPa), and conveying water at 10 fps (3 m/s); the PxA force on the elbow is 207 lbf (920 N) whereas the force due to momentum is only 4.5 lbf (20 N).

## 5.0 SPECIFIC GUIDELINES

### 5.1 Cavitation Does Not Exist

- a. Criteria
  - i. Transient pressures never reach vapor pressure anywhere in the model
- b. Recommended Actions
  - i. Use force pair results with the safety factor determined in the CAC (1.25:1). Prepare a force vs. time plot for use in the calculation report.
  - ii. Are force pair results to be used only for the specific location where they are specified in the HTA software?
    1. Yes – Forces are provided at all changes in direction that are of interest to Pipe Stress Engineers. No further action is needed.
    2. No – For simplicity or due to the preliminary state of the piping design, force pairs have only been defined at key representative locations. Additional diligence is required.
      - a. Is the maximum force equivalent to (within 5%) the maximum pressure differential times area?
        - i. Yes – Use force pair results with the safety factor previously specified. No further action is needed.
- ii. No – Review the HTA software output pressure vs. length profile at particular times of interest and look for severe pressure spikes due to constructive transient wave interference that might occur close to, but outside of, the force pairs. Note that HTA software animation features (see Part 2 Definitions above) can simplify the identification of the constructive wave interference.
  1. Do such pressure spikes occur?
    - a. No – Use force pair result with the safety factor previously specified. No further action is needed.
    - b. Yes – Select a time step for constructive wave interference evaluation (e.g., see Fig. 1). Determine the pressures at the peak and minimum of the transient wave. Subtract the minimum from the peak pressure and multiply the difference by the pipe's inside cross-sectional area to calculate a maximum envelope HTF. Multiply the calculated HTF by the safety factor previously determined for pressure by the CAC (Part 1, Stewart et al. [1]). Recommended best practice is to retain the selected time step for constructive wave interference (as shown in Fig. 1) pressure vs. length profile graph and include it in the report with a force-time history graph. Explain that other elbows in that vicinity should consider forces of the calculated magnitude.

## 5.2 Limited or Major Cavitation Exists, but Max Forces Occur before Cavitation Occurs

- a. Apply safety factors to forces calculated before the onset of cavitation based on Step 5.1.
- b. Apply safety factors to forces calculated after the onset of cavitation based on Step 5.3 or 5.4 below.
  - i. Are calculated forces before cavitation larger than forces after cavitation?
    1. Yes – Report forces before cavitation according to Step 5.1. The forces after cavitation may be ignored.
    2. No – Report forces after cavitation according to Steps 5.3 or 5.4. The forces before cavitation may be ignored.

## 5.3 Limited or Major Cavitation Exists in the Model, but it Occurs in Part of the System Isolated by a Closed Valve or Equivalent

- a. Criteria
  - i. Transient pressures never reach vapor pressure in a hydraulically isolated (e.g., valved off) portion of the system.
  - ii. Transient pressures do reach vapor pressure in a hydraulically isolated part of the system, but only after the isolation is completed (e.g., valve completely closed).
- b. Recommended Actions
  - i. For the part of the system not experiencing cavitation, use Step 5.1.
  - ii. For the part of the system experiencing cavitation, use Step 5.4 below.

## 5.4 Limited or Major Cavitation Exists

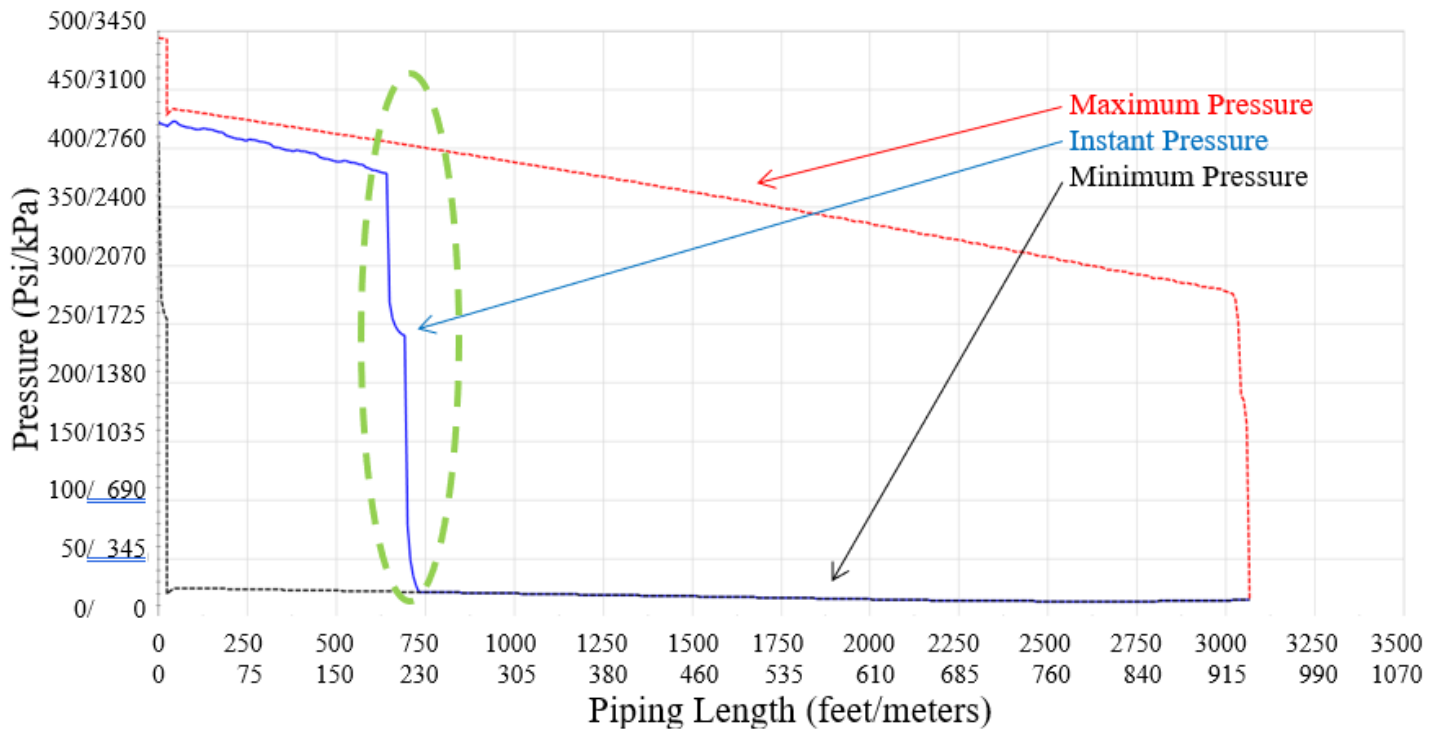
- a. Criteria
  - i. Cavitation volumes ratios are all below 100% of computing volume and are acceptable per the CAC ( $CVR_{MAX} < 100\%$ , see Part 1, Stewart et al. [1]).
- b. Recommended Actions
  - i. Apply a CSF one level higher than the safety factor determined in the Part 1 CAC to the force pair results then go to Step 5.1.b.ii above.
  - ii. Are the forces accepted by the Pipe Stress Engineer?
    1. Yes – Conclude

2. No – Are there pressure spikes due to numerical model noise that can be filtered out (discarded) of the force results?
  - a. Yes – Filter out (discard) the numerical model noise related pressure spikes and apply CSF as recommended above. Are the forces accepted by the Pipe Stress Engineer?
    - i. Yes – Conclude
    - ii. No – Manually assess maximum envelope force so that more conservative and reliable results may be used with a lower safety factor which can result in a lower calculated force. Review the HTA software pressure vs. length profile at times of interest and look for the largest, sharpest transient waves moving through the system. Refer to Fig. 1. Select the worst case instant time as in Fig. 1. Measure the pressures at the peak and minimum of the transient wave. Subtract the minimum from the peak pressure and multiply the difference by the pipe's inside cross-sectional area to calculate a maximum envelope HTF. Multiply the HTF by the same safety factor previously determined for pressure by the Part 1 CAC (do not apply the next higher safety factor). Recommended best practice is to retain the selected time step for instant pressure (as shown in Fig. 1) and include it as a graph in the calculation report. Report the calculated maximum envelope force with the plot as shown in Fig. 1 Calculation Note. Again, ignore spikes of one or a few time steps per the criteria above. Note that HTA software animation features (see Part 2 Definitions above) can simplify the identification of the preferred time step in Fig. 1 for the sharpest and largest transient wave.
  - b. No – follow the exact same steps as described in the preceding section 5.4.b.ii.2.a.ii (“No”)

## 5.5 Extreme Cavitation Exists

- a. Criteria
  - i. Some cavitation volumes are greater than 100% of computing volume ( $CVR_{MAX} > 100$ , see Part 1, [1])
- b. Recommended Actions
  - i. Systems with cavitation volumes greater than 100% are not acceptable. Evaluation of HTFs is not required.

This concludes the guideline. See Discussion section below for background information on the guideline.



Calculation note: Differential pressure is 360 psi (2,480 kPa); In a 2.067 in (5.25 cm) inside diameter pipe  $\Delta P \times A = 1,210 \text{ lbf}$  (5.37 kN)

**Figure 1: Example max/min and instant pressure profile (at a particular time of interest) indicating a sharp transient wave and recommended calculation note to accompany the graph**

## DISCUSSION

The choice of appropriate safety factors for imbalanced forces during cavitation was based on three competing factors:

1. Imbalanced force prediction during cavitation is even more uncertain than pressure predictions. Thus, in general, a larger safety factor should be used.
2. Different methods of determining a maximum force are specified in the guideline – some are more conservative and some less.
3. It did not seem reasonable to use increased safety factors in some cases where a more conservative method of maximum force was used, especially if the method is not affected by wave timing error (point force or manual assessment methods).

Therefore, decisions were made in some cases to increase safety factors up one level (see defined levels in this Part 2, Section 3, Safety Factors). In other cases it was decided not to increase safety factors up a level.

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