Technical Paper

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A COMPREHENSIVE DISCUSSION OF SONIC CHOKING IN PIPE SYSTEMS FOR STEADY, COMPRESSIBLE FLOW

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ABSTRACT

Understanding and predicting sonic choking in pipe network systems can be critically important. Among the reasons is to ensure safety and to improve the performance of the system. In some cases, sonic choking is the dominant aspect of the system behavior and there is no way to understand the system without understanding sonic choking and where it occurs. However, sonic choking is (in general) poorly understood and (at best) incompletely documented in the literature. The literature often provides misleading examples built on unstated (and unrealistic) assumptions. Some situations where sonic choking occurs are almost completely ignored in the literature. The purpose of this paper is to provide a comprehensive reference for practicing engineers for all possible sonic choking behavior in any generalized, pipe network system with superheated, steady-state, single-phase gas flow. Discussions include multiple choking points in series and in parallel, as well as in systems with real gas behavior experiencing heat transfer in non-horizontal pipes.

KEYWORDS:

Gas dynamics, compressible flow, sonic choking, critical flow, Mach number, pipe flow

NOMENCLATURE

A	cross-sectional area (ft^2 / m^2)
c	acoustic (sonic) velocity (ft/s / m/s)
C_D	discharge coefficient (-)
D	inner diameter (ft / m)
f	friction factor, Darcy-Weisbach (-)
L	length (ft / m)
$\dot{m}$	mass flowrate (lbm/s / kg/s)
M	Mach number (-)
P, P_o	static and stagnation pressure (psi / kPa)
R	gas constant (Btu/lbm-R / J/kg-K)
s	entropy (Btu/R / J/K)
T, T_o	static and stagnation temperature (deg F / deg C)
V	fluid velocity (ft/s / m/s)
Z	compressibility factor, correction for non-ideal gas (-)
γ	isentropic expansion coefficient (-)
ρ	static density (lbm/ft^3 / kg/m^3)

1. INTRODUCTION

Sonic choking is a phenomenon that occurs in pipe systems which involves the local bulk velocity of the gas/steam reaching the sonic velocity. When this happens, the flow capacity of the system reaches a maximum for the given supply conditions. A somewhat similar mechanism occurs for multi-phase flow, but the discussion here will focus on steady-state, superheated, single-phase gaseous conditions.

The principles discussed in this paper apply to all pipe system behavior including real gas behavior with heat transfer and elevation changes (more relevant for dense/high pressure gases). However, in order to better explain the various principles, focus will be given to ideal gases under adiabatic conditions in horizontal pipe systems. This allows closed-form, analytic methods to be used for computational examples while still retaining the important physical principles being explained. Including real gas behavior, heat transfer and elevation changes can be achieved, but this requires capable computational software designed to properly consider sonic choking in complicated pipe network systems. While this is beyond the scope of this paper, engineers will be alerted to what to look for when evaluating such software.

Most discussions of sonic choking focus on a single flow path. This may be a single pipe or two (or more) pipes connected in series. This leaves open the question of what happens in complicated pipe networks. This paper will discuss:

- Situations with
    - pipe networks upstream and/or downstream of a choke point
    - multiple choke points in series flow paths
    - multiple choke points in parallel flow paths
- Which sonic choking point controls the flowrate
- How to determine flow and thermodynamic conditions downstream of the choke point (and why this is important)

2. SONIC VELOCITY

To avoid any misunderstanding, the terms “sonic velocity”, “acoustic velocity” and “speed of sound” are all synonymous. The first published predictive equation for the speed of sound was made by Newton, 1687, [1]. Newton’s equation was subsequently corrected from being based on an isothermal process to an isentropic process. Laplace, in the early 1800’s, is usually given credit for this (Anderson, 2004, [2]).

The speed of sound, c, is given by the following (and valid for real gases):

$$c^2 = \left.\frac{\partial P}{\partial \rho}\right|_s \tag{1}$$

Using isentropic relationships one can also determine the speed of sound (still valid for real gases) as:

$$c = \sqrt{\frac{\gamma P}{\rho}} \tag{2}$$

Using a real gas equation of state, Eq. 2 can be rendered:

$$c = \sqrt{Z\gamma RT} \tag{3}$$

Eqs. 1-3 can be found in any compressible flow textbook, e.g., Anderson, 2004, [2], pp. 47-48.

3. THREE GEOMETRIES WHERE SONIC CHOKING CAN OCCUR

The mechanism of steady-state, sonic choking is the same no matter where it occurs in a pipe system. Specifically, if the flow in the pipe system results in a local bulk velocity which reaches the sonic velocity, the flow will choke in that location. There is no limit to the number of choke points in a pipe system other than the number of geometric locations in that system where sonic choking can occur. There are three geometric situations where sonic choking can occur (Walters, 2000, [3]):

3.1 Restriction Choking

Restriction Choking occurs in a constant diameter pipe where there is some local reduction in flow area such as through an orifice as in Fig. 1. Here the reduction in area causes a local velocity increase and, at the smallest fluid dynamic diameter, the flow reaches sonic velocity and choking occurs. In the case of an orifice, the vena contracta that occurs means the smallest effective flow area will be less than the physical orifice area. The ratio of the smallest effective flow area to the physical area is frequently referred to as the *discharge coefficient*, C_D. While Fig. 1 shows the choke point right at the orifice, because of the vena contracta, wall separation and turbulence, this choke point may physically be downstream of the orifice.

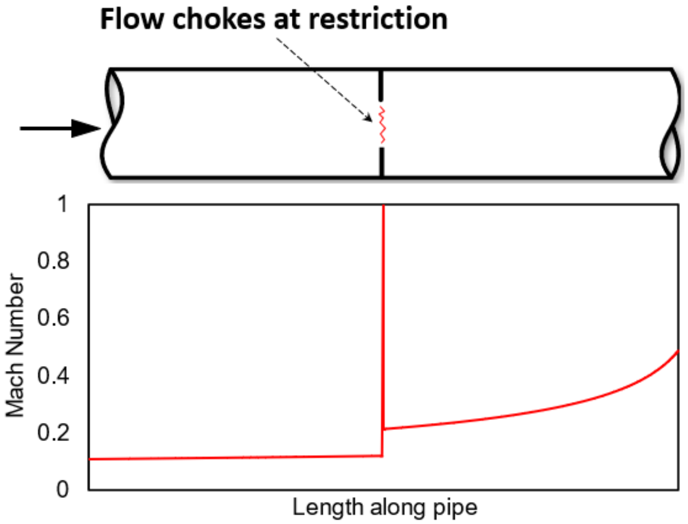

FIGURE 1: RESTRICTION CHOKING IN A CONSTANT DIAMETER PIPE AND EXAMPLE MACH NUMBER VARIATION

3.2 Endpoint Choking

Endpoint Choking occurs at the end of a constant diameter pipe as it discharges into a large volume such as a vessel or the atmosphere as in Fig. 2. Here, the gas velocity increases as the gas travels from the entrance to the exit and it reaches the sonic velocity at the pipe exit. In steady-state flow it is not possible for the flow to reach sonic velocity in the middle of a constant diameter pipe.[1] Here the Mach number asymptotically reaches 1 at the end of the pipe.

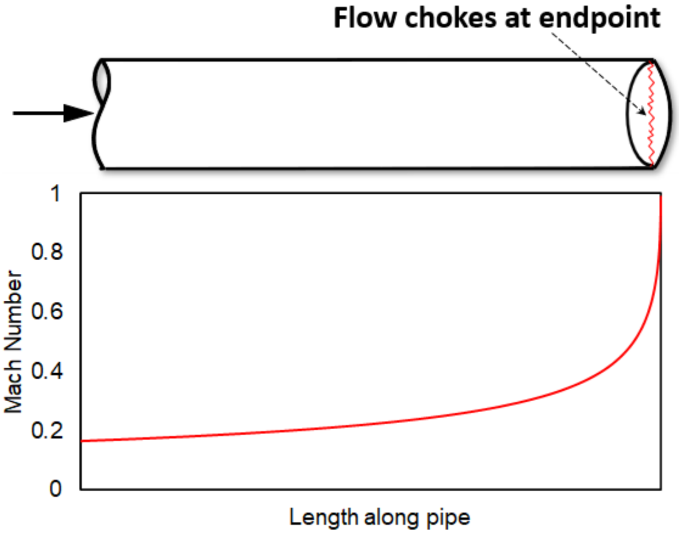

FIGURE 2: ENDPOINT CHOKING IN A CONSTANT DIAMETER PIPE AND EXAMPLE MACH NUMBER VARIATION

3.3 Expansion Choking

Expansion choking occurs at the end of a constant diameter pipe where it transitions to a total flow area larger than that of

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[1] This is true for adiabatic flow and for most typical flow conditions. However, it is possible to apply non-uniform, localized cooling to the pipe and cause sonic choking to occur in the middle of a constant diameter pipe. It is also possible for a localized vena contracta and/or wall separation in the middle of a constant diameter pipe to create a sonic choking point as discussed later in Section 3.4.

the pipe. This can occur, for example, at an expansion pipe (as shown in Fig. 3) or a tee where the downstream pipe area exceeds that of the incoming pipe. Another variation on this choke point geometry is the exit of a pipe into a large header pipe. The larger area in the header pipe makes the smaller connecting pipe susceptible to sonic choking. Note that Endpoint Choking is a special case of Expansion Choking where the downstream flow area is effectively infinite rather than finite.

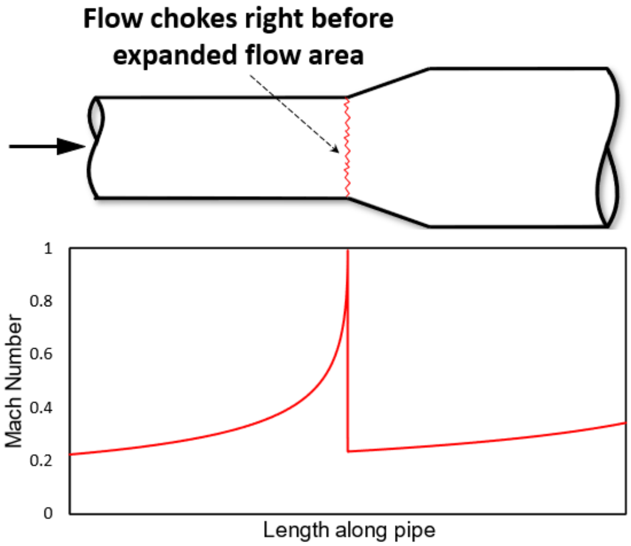


FIGURE 3: EXPANSION CHOKING IN A PIPE WHERE THE TOTAL FLOW AREA INCREASES

3.4 Some subtle exceptions

Real world fluid dynamic behavior such as a vena contracta and/or wall separation can create locations of effective flow area reduction. This can result in sonic choking similar to Restriction Choking (Section 3.1) even if no geometric restriction exists. Examples include flow through a reducer and flow in a constant diameter elbow which experiences wall separation.

4. MASS FLOWRATE AT CHOKE POINT

Saad, 1993, [4], p. 97, develops a general mass conservation equation in the following form (with a real gas compressibility factor, Z, added by the author to make the equation applicable for real gases):

$$\dot{m} = A\frac{P_o}{\sqrt{T_o}}\sqrt{\frac{\gamma}{ZR}}\,M\left(1+\frac{\gamma-1}{2}M^2\right)^{-(\gamma+1)/[2(\gamma-1)]} \qquad (4)$$

Choked flow occurs when $M = 1$, and using that in Eq. 4 obtains:

$$\dot{m}_{choked} = A\frac{P_o}{\sqrt{T_o}}\sqrt{\frac{\gamma}{ZR}}\left(1+\frac{\gamma-1}{2}\right)^{-(\gamma+1)/[2(\gamma-1)]} \qquad (5)$$

Eqs. 4-5 are particularly useful for assessing mass conservation and choked flowrate in pipe systems.

5. SIMPLIFIED LUMPED ADIABATIC EQUATION WITH FRICTION IN SINGLE PIPE

A well-known closed-form solution to predict compressible flow in a horizontal, constant diameter pipe under adiabatic flow conditions with constant friction is (Saad, 1993, [4], p. 209):

$$\frac{fL}{D} = \frac{1}{\gamma}\left(\frac{1}{{M_1}^2} - \frac{1}{{M_2}^2}\right) + \frac{\gamma+1}{2\gamma}\ln\left[\left(\frac{{M_1}^2}{{M_2}^2}\right)\left(\frac{1+\frac{\gamma-1}{2}{M_2}^2}{1+\frac{\gamma-1}{2}{M_1}^2}\right)\right] \qquad (6)$$

Using iteration, Eq. 6 can predict exit conditions (at subscript 2) for given inlet conditions (subscript 1) for a given friction factor, *f*, and *L*/*D* ratio. It is common to see solutions to Eq. 6 presented in compressible flow textbooks in table form (e.g., Anderson, 2004, [2], Appendix Table A.4 and Saad, 1993, [4], Appendix Table A4).

In real systems with pipe networks, real gas behavior, heat transfer, variable friction and elevation changes, one often needs to account for more complex behavior than possible with Eq. 6. Even so, Eq. 6 is useful for demonstrating important principles of choked flow. In this spirit, we will use Eq. 6 for a few examples.

6. SINGLE PIPE AND SEQUENTIAL PIPE EXAMPLES WITH ONE CHOKE POINT

To solve flow in a single, constant-diameter pipe or a sequence of connected pipes (possibly with varying diameters), we need to know boundary conditions. There are essentially three possible combinations of upstream/downstream conditions as shown in Fig. 4.

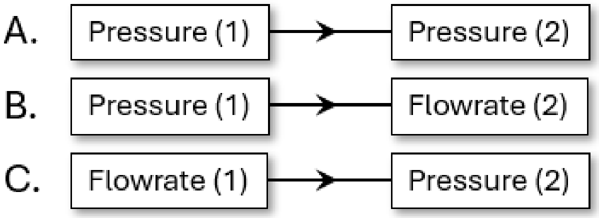


FIGURE 4: THREE TYPES OF SOLVABLE BOUNDARY CONDITION COMBINATIONS FOR A PIPE OR PIPE SEQUENCE (FLOW FROM LEFT TO RIGHT)

Note that the flow direction in Fig. 4 is known for Cases B and C only. The Case A flow direction depends on the pressures at points 1 and 2 (Fig. 4 assumes point 1 pressure is higher than point 2). Each case will also require a known temperature at the inflow boundary. Temperature at the outflow boundary is not required as it will not influence the flow solution in the pipe.

All worked examples will use air and assume it is an ideal gas and calorically perfect with an isentropic expansion coefficient of 1.4. Entrance losses to the supply pipe are ignored. Pipe exits are assumed to be to infinite volumes which are quiescent other than the pipe exit air jet. All pipes and components are adiabatic. Excel spreadsheets are provided for each example in Walters, 2024, [5] with one exception. The final example in Section 8 is too complicated for a spreadsheet approach. In that case, a solution was obtained using a commercially available software package, *AFT Arrow,* [6].

Fig. 5 gives geometric and boundary condition data for the examples which follow.

Input	English Units	Metic Units
Pipe data		
Diameter (P1, P2)	3 in.	7.62 cm
Length (P1)	100 ft	30.5 m
Friction Factor (P1, P2)	0.017	0.017
Length (P2)	25 ft	7.6 m
Diameter (P3)	4 in.	10.16 cm
Length (P3)	25 ft	7.6 m
Friction Factor (P3)	0.016	0.016
Junction boundary data		
Pressure (Stag.) Inlet (J1)	400 psia	2758 kPa
Temperature (Stag.) Inlet (J1)	200 F	93.3 C
Orifice C_DA (J2, J12, J22)	2 sq. in	12.9 sq. cm

FIGURE 5: COMMON DATA USED IN EXAMPLES

6.1 Endpoint Choking (Example 6.1)

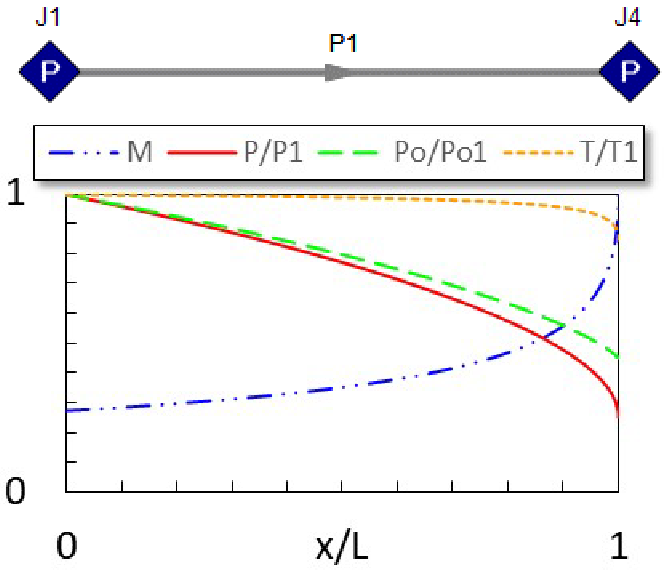
The most straightforward application of Eq. 6 is for Endpoint Choking (Fig. 2). The solution follows the Fig. 4a combination of boundary conditions. Fig. 6 shows an example with results obtained using Eq. 6. Output results show that choking will occur for any exit static pressure at J4 of 95.3 psia (657.1 kPa) or below. As the input here was 80 psia (551.6 kPa) at J4, sonic choking occurred at the end of Pipe P1.

For adiabatic flow with a calorically perfect gas, we know T_o at the outlet of Pipe P1 (it is the same as the inlet). Further, for the simplified case here assuming Pipe P1 is adiabatic, we can use Eq. 6 to determine M_2 at the end of Pipe P1. Using standard relationships, this allows us to determine P_o.

When one is dealing with a real gas then one cannot know T_o at the end of Pipe P1. Further, with a real gas, elevation changes, heat transfer and/or variable friction, one cannot use Eq. 6. So, there is no way to find P_o at the end of Pipe P1. Thus, an accurate solution even for the normally straightforward case of Endpoint Choking gets quite complicated. How then can this situation be predicted? The only way is to use the full conservation equations and guess a choked flowrate – and then to iterate heavily to the conditions at the end of Pipe P1[2]. For more on this see Walters, 2000, [3].

6.2 Expansion Choking (Example 6.2)

Building on Example 6.1, here an expansion from 3 to 4 inches (7.6 to 10.2 cm) occurs followed by a second pipe (see Fig. 7). Pipe P1 has the same data as in Example 6.1. Output results are shown in Fig. 7. Recall that Expansion Choking occurs at the end of the pipe with the smaller area (Fig. 3). Here choking occurs at the end of Pipe P1 and, hence, results for P1 are the same as in Example 6.1 (since all the Pipe P1 and J1 data is the same).



Input	English Units	Metic Units
Pipe 1, Jct 1 - see Fig. 5		
Pressure (Static) Outlet (J4)	80 psia	551.6 kPa
Output (Pipe 1)		
Mass flow rate	26.40 lbm/s	11.97 kg/s
Mach number Inlet	0.273	0.273
Pressure (Static) Inlet	379.8 psia	2618.9 kPa
Temperature (Static) Inlet	190.3 F	88.0 C
Velocity Inlet	341.0 ft/s	104.0 m/s
Mach number Outlet	1.0	1.0
Pressure (Stag.) Outlet	180.4 psia	1243.8 kPa
Pressure (Static) Outlet	95.3 psia	657.1 kPa
Temperature (Static) Outlet	90.1 F	32.3 C
Velocity Outlet	1149.4 ft/s	350.4 m/s

FIGURE 6: EXAMPLE 6.1 OF ENDPOINT CHOKING

How are Pipe P3 results obtained? How is the pressure drop across the J3 expansion calculated? There is no formula for calculating the pressure drop at J3. Once choking occurs at the J3 expansion then calculations of Pipe P3 have to pivot to a different strategy.

What is known in Pipe P3? Two things are known: 1) from mass conservation the mass flow must be the same as Pipe P1 and, 2) from energy conservation the stagnation enthalpy at the *outlet* of Pipe P1 must be the same as the *inlet* of Pipe P3. In other words, the sonic choking process across J3 is adiabatic. See Walters, 2000, [3], Eqs. 4 and 8. For the calorically perfect gas assumed here, the stagnation temperature is also constant across J3. With these two knowns, one can solve Pipe P3 essentially the same as Fig. 4c. In Fig. 4c the upstream flowrate is known (it is the choked flowrate from Pipe P1) and the downstream static pressure is known (at J4 it is 100.6 psia / 693.5 kPa, see Fig. 7).

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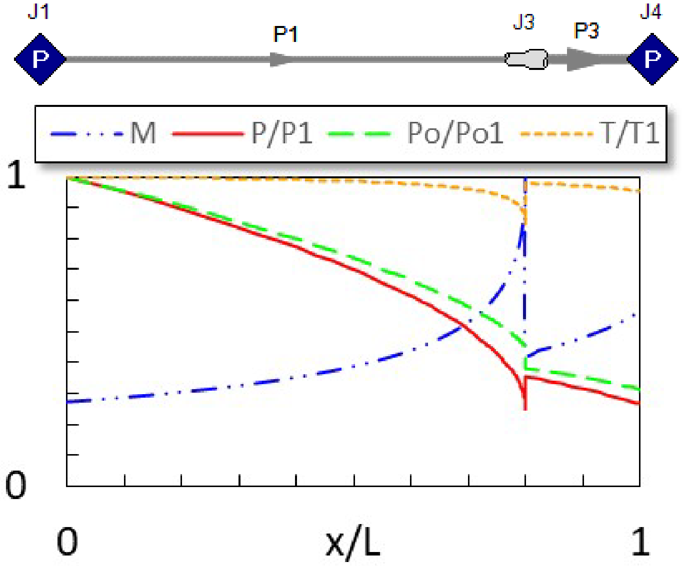
[2] When using steady flow calculation techniques. In principle, a transient compressible flow analysis could also be used where the transient solver is allowed to run until steady conditions are established.

In our example the conditions are adiabatic everywhere which means the stagnation enthalpy is constant everywhere. Again, with the calorically perfect gas assumption, the stagnation temperature is also constant everywhere. Were heat transfer to occur on either or both Pipes P1 and P3, then the stagnation enthalpy and temperature will change along each pipe. But the behavior across J3 will be the same – known mass flowrate and constant stagnation enthalpy and temperature.

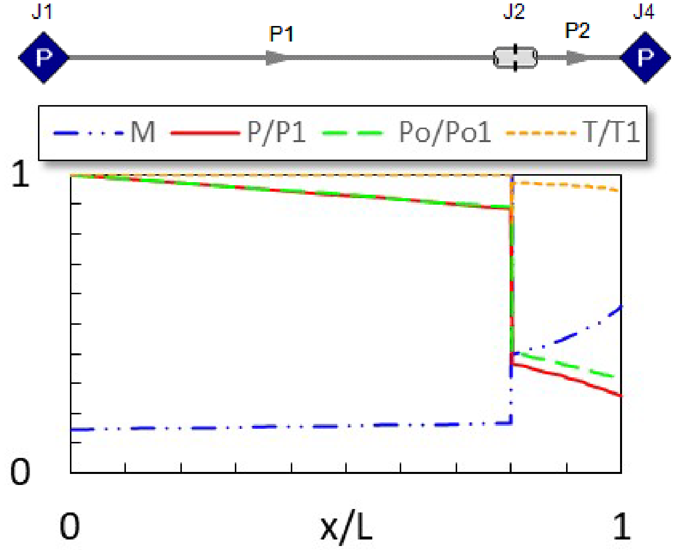


Input	English Units	Metic Units
Pipes 1 & 3, Jct 1 - see Fig. 5		
Pressure (Static) Outlet (J4)	100.6 psia	693.6 kPa
Output (Pipe 1)		
Same as Example 6.1, Pipe 1		
Output (Pipe 3)		
Mach number Inlet	0.429	0.429
Pressure (Stag.) Inlet	152.6 psia	1052.1 kPa
Pressure (Static) Inlet	134.5 psia	927.1 kPa
Temperature (Static) Inlet	176.6 F	80.3 C
Velocity Inlet	530.3 ft/s	161.7 m/s
Mach number Outlet	0.566	0.566
Pressure (Stag.) Outlet	125.0 psia	861.8 kPa
Pressure (Static) Outlet	100.6 psia	693.5 kPa
Temperature (Static) Outlet	160.3 F	71.3 C
Velocity Outlet	690.8 ft/s	350.4 m/s

FIGURE 7: EXAMPLE 6.2 OF EXPANSION CHOKING

6.3 Restriction Choking (Example 6.3)

This choking situation is altogether different from the first two. Now choking occurs in a pipe sequence of constant diameter where there is a restriction (Fig. 8). Recall that Restriction Choking occurs at a local area reduction (Fig. 1). Fig. 8 shows that Pipe P1 has the same input data as in Examples 6.1 and 6.2. But now there is an orifice at J2 with a C_DA of 2 in^2 (12.9 cm^2). This represents a sharp-edged orifice with a diameter of 2.06 inches (5.23 cm) with a C_D of 0.6 inside of a 3-inch (7.6 cm) pipe. In general, restriction choking can occur regardless of the downstream pipe diameter, Pipe P2.



Input	English Units	Metic Units
Pipes 1 & 2, Jcts 1 & 2 - see Fig. 5		
Pressure (Static) Outlet (J4)	101.1 psia	697.1 kPa
Output (Pipe 1)		
Mass flow rate	14.74 lbm/s	6.69 kg/s
Mach number Inlet	0.148	0.148
Pressure (Static) Inlet	394.0 psia	2716.2 kPa
Temperature (Static) Inlet	197.1 F	91.7 C
Velocity Inlet	185.5 ft/s	56.5 m/s
Mach number Outlet	0.166	0.166
Pressure (Stag.) Outlet	355.9 psia	2454.2 kPa
Pressure (Static) Outlet	349.1 psia	2407.1 kPa
Temperature (Static) Outlet	196.4 F	91.3 C
Velocity Outlet	209.0 ft/s	63.7 m/s
Output (J2 at Min. Flow Area)		
Mach number	1.0	1.0
Pressure (Stag.)	355.9 psia	2454.2 kPa
Pressure (Static)	188.0 psia	1296.5 kPa
Temperature (Static)	90.1 F	32.3 C
Velocity	1149.4 ft/s	350.4 m/s
Output (Pipe 2)		
Mach number Inlet	0.396	0.396
Pressure (Stag.) Inlet	161.4 psia	1112.6 kPa
Pressure (Static) Inlet	144.8 psia	998.4 kPa
Temperature (Static) Inlet	179.9 F	82.2 C
Velocity Inlet	491.3 ft/s	149.8 m/s
Mach number Outlet	0.559	0.559
Pressure (Stag.) Outlet	125.0 psia	861.8 kPa
Pressure (Static) Outlet	101.1 psia	696.9 kPa
Temperature (Static) Outlet	161.1 F	71.7 C
Velocity Outlet	683.3 ft/s	350.4 m/s

FIGURE 8: EXAMPLE 6.3 OF RESTRICTION CHOKING

Here conditions are such that the Mach number reaches 1 at the minimum effective flow area (the C_DA) of the orifice. How are Pipe P2 results obtained? How is the pressure drop across the J2 restriction calculated? It is in fact the same strategy as for Example 6.2. Similar to Example 6.2, there is no formula for calculating the pressure drop at J2. One solves Pipe P2 using the Fig. 4c approach – a known upstream flowrate (it is the choked flowrate at the J2 orifice) and the downstream static pressure is known (at J4, 101.1 psia / 697.1 kPa).

When applying Eq. 5 at an orifice, the appropriate area to use is the C_DA. Frequently in the literature, Restriction Choking is solved assuming the stagnation pressure and temperature at an orifice (e.g., J2) is the same as that to the inlet pipe (e.g., J1). Thus, Eq. 5 would be employed using Pipe P1 *inlet* conditions – not the *outlet*. In essence, this assumes that Pipe P1 is frictionless and horizontal. Further, any stagnation temperature change in Pipe P1 is also neglected. This is only true for adiabatic conditions in a horizontal pipe with a calorically perfect gas.

In summary, it is essential to understand here that the only way to predict the choked flow conditions at J2 is to determine the local stagnation pressure and temperature *at the orifice* and use Eq. 5. Similar to the discussion in Example 6.1, the only way to determine these conditions is to iterate heavily.

6.4 Summary of Steps to Solve Examples 6.2 and 6.3

In solving sonic choking in a connected pipe sequence, the pipes must be hydraulically decoupled and each solved with a different strategy of boundary conditions. Gathering the discussion of Examples 6.2 and 6.3, here is a summary of steps:

1. Solve Pipe P1 using the Fig. 4a boundary condition combination
2. Determine if Pipe P1 is choked and, if so, the choked flowrate using Eq. 5
3. Solve the downstream Pipe P2/P3 using the Fig. 4c boundary condition combination with the known choked flowrate as the upstream boundary condition
    a. The thermal boundary condition at the upstream of Pipe P2/P3 derives from an energy balance of the choking point – the stagnation enthalpy is constant across J2/J3

6.5 Importance of Accurately Calculating Conditions Downstream of Sonic Choke Points

Once sonic choking is identified and the sonically choked flowrate is determined, the engineer's task is complete, isn't it? In some cases, the answer to this question is "Yes". But in other cases, the answer is emphatically "No". Why would an engineer be interested in knowing more about the pipe system than the sonically choked flowrate? And why are conditions downstream of choke points important? Here are a number of reasons why an engineer may want to know more than just the choked flowrate:

- Pressure
    - Maximum for adequate pipe strength
    - Minimum to avoid condensation
- Temperature
    - Maximum for pipe surface temperature safety and need for insulation (so workers do not get burned)
    - Minimum to avoid condensation
    - Actual temperature of system outlet locations to meet process delivery condition requirements
- Velocity
    - Noise prediction
    - Thrust prediction for
        - aerospace systems
        - structural supports for discharging pipes such as relief piping
- Density
    - Thrust prediction for aerospace systems (calculation of density specific impulse)
- Flowrate distribution
    - When there are multiple system discharge points one often wants to know the flowrate to each one
- Sonic flow capacity
    - Downstream conditions can eliminate upstream choke points and reduce flow capacity
- All conditions at all locations
    - Needed in order to initialize a transient analysis of the system (e.g., Schroeder, et. al., 2017, [7], which used the Ref. [6] commercial software for this purpose)

7. MULTIPLE CHOKING POINTS IN SERIES – SEQUENTIAL PIPES

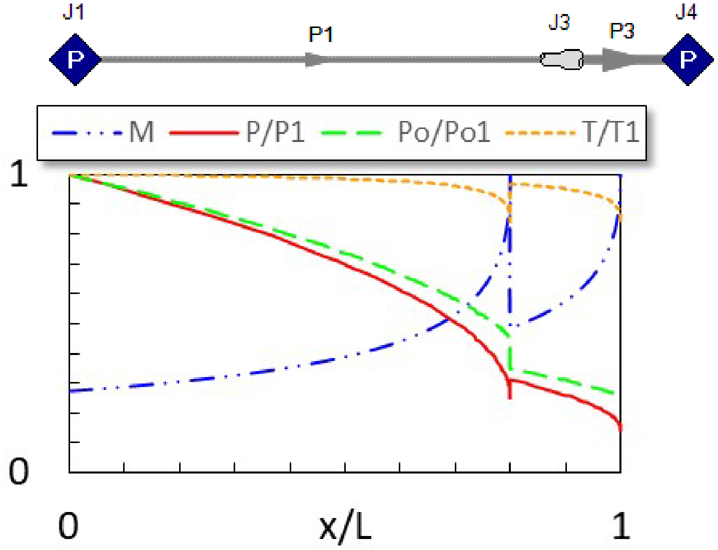
If choked flow means that the maximum flow exists for given supply conditions, how can more than one choke point exist in a connected pipe sequence? Is that even possible? In fact, it is quite possible.

7.1 Two Choking Points in Series With Expansion Choking (Example 7.1)

One can see this using Example 6.2 on Expansion Choking. Look at Fig. 7 and the Pipe P3 outlet static pressure. It is fixed at 100.6 psia (693.5 kPa). What if this pressure was lowered?

Fig. 9 shows the results. The only change here is the lowering of the pressure at J4 to 50 psia (344.7 kPa). All results are the same (as Example 6.2) in Pipe P1. The only change in results is in Pipe P3. If one looks closely at the outlet static pressure one can see it is 53.6 psia (369.6 kPa). This is the static pressure at J4 below which sonic choking will occur at the end of Pipe 3. The pressure used here of 50 psia (344.7 kPa) is below this value (hence, the second choke point). The second choke point in Pipe P3 is Endpoint Choking. It is thus clear that two different types of choking can occur in the same pipe sequence.

What is the effect of the second sonic choking point? First, it is clear it has no impact on anything upstream of the first choke point. It does not change the choked flowrate. What it does (comparing results in Fig. 7 to Fig. 9) is *redistribute* the pressure and all flow conditions in Pipe P3.



Input	**English Units**	**Metic Units**
Pipes 1 & 3, Jct 1 - see Fig. 5		
Pressure (Static) Outlet (J4)	50 psia	344.7 kPa
Output (Pipe 1)		
Same as Example 6.1, Pipe 1		
Output (Pipe 2)		
Mach number Inlet	0.485	0.485
Pressure (Stag.) Inlet	139.0 psia	958.5 kPa
Pressure (Static) Inlet	118.4 psia	816.1 kPa
Temperature (Static) Inlet	170.4 F	76.9 C
Velocity Inlet	596.6 ft/s	181.9 m/s
Mach number Outlet	1.0	1.0
Pressure (Stag.) Outlet	101.5 psia	699.6 kPa
Pressure (Static) Outlet	53.6 psia	369.6 kPa
Temperature (Static) Outlet	90.1 F	32.3 C
Velocity Outlet	1149.4 ft/s	350.4 m/s

FIGURE 9: EXAMPLE 7.1 OF EXPANSION CHOKING WITH TWO CHOKING POINTS IN SERIES

There is an important conclusion to be drawn here. When there are multiple choke points in series, the first choke point controls the flowrate. The remaining choke points merely redistribute the pressure and other flow conditions downstream of the first choke point.

Both choke points honor Eq. 5. With the calorically perfect gas assumption and adiabatic flow, T_o is constant throughout and thus the same value at the outlet of Pipe P1 and P3. However, P_o is different and must be determined at each choke point location. That means that once one knows the first choke point conditions, one can no longer count on being able to use that information downstream of the choke point – except for applying mass and energy conservation (constant stagnation enthalpy) across the first choke point.

In our case, determining P_o at the outlet of Pipe P3 is trivial using Eq. 5 as explained in the following. Armed with a known flowrate (the choked flowrate from the first choke point in Pipe P1), and also known T_o, the P_o can be solved for directly in Eq. 5. In a more general case, T_o is not known at the outlet of Pipe P3 and thus both T_o and P_o can only be determined by heavy iteration when using steady flow relationships (Walters, 2000, [3]).

Note that there is a static pressure at J4 above which no choking occurs anywhere. This is hard to determine precisely because of the asymptotic behavior at the exit of Pipe P1, but it is roughly 126 psia (869 kPa). This result is not shown here for brevity.

7.2 Two Choking Points in Series With Restriction Choking (Example 7.2)

The easiest place to see this is in Example 6.3 on Restriction Choking. Similar to Example 7.1, we will reduce the static pressure at J4 from 101.1 to 50 psia (697.1 to 344.7 kPa).

Fig. 10 shows results. Similar to Example 7.1, all results are the same in Pipe P1 and at the J2 restriction. The only change in results is in Pipe P2.

The behavior in Pipe P2 is otherwise the same as in Example 7.1 as summarized below:

- The Pipe P2 outlet static pressure is 53.2 psia (367 kPa) which is the pressure at J4 below which sonic choking will occur at the end of Pipe P2
    - The static pressure assumed here at J4 is 50 psia (345 kPa) and happens to be just below this value
- The second choke point does not change the flowrate, it just redistributes the pressure and all flow conditions in Pipe P2
- The first choke point controls the flowrate for the entire pipe sequence
- Both choke points honor Eq. 5
- For our case of adiabatic flow and calorically perfect gas, predicting P_o at the outlet of Pipe P2 is trivial using Eq. 5

7.3 Three Choking Points in Series (Example 7.3)

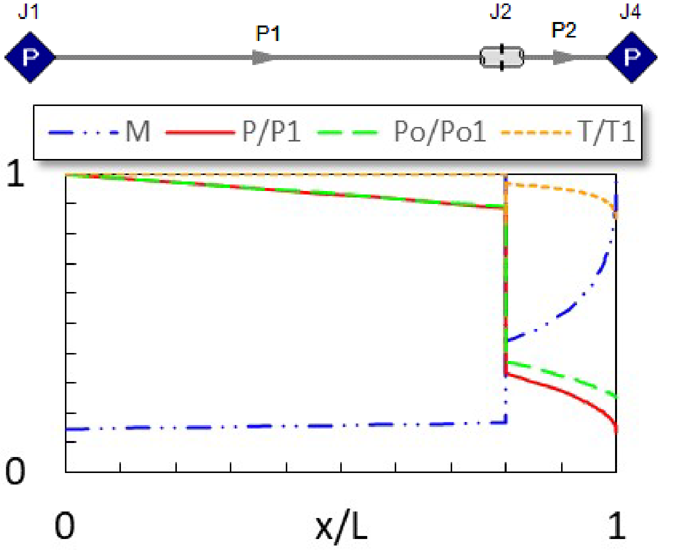
Here each geometric type of sonic choking from Section 3 happens in the same sequence of connected pipes. See Fig. 11. The first choke point is Restriction Choking at J2. Then Expansion Choking occurs at the J3 expansion from 3 to 4-inch (7.6 to 10.2 cm) diameter pipe. Finally, Endpoint Choking occurs at the end of Pipe P3 where it connects to J4. Results are shown in Fig. 11.

The strategy for solving Pipe P3 is the same as Section 6.4 Step 3. The stagnation enthalpy and mass flowrate across J3 are constant and Pipe P3 is solved using the Fig. 4c boundary condition combination.

Similar to Examples 7.1 and 7.2, increasing the J4 outlet pressure will eventually result in no choking. From Fig. 11 one can see the Pipe P3 outlet static pressure of 29.9 psia (206 kPa). In this example, that is the pressure above which eliminates

Endpoint Choking. But there are still two other choking points. Each has an outlet pressure at J4 above which the choke point will go away. By trial and error, it can be determined that these are the static pressures at J4 that will eliminate the choke points:

- Above ~30 psia (~207 kPa) the Endpoint Choking at J4 will become unchoked
- Above ~70 psia (~483 kPa) the Expansion Choking at J3 will become unchoked
- Above ~169 psia (~1165 kPa) the Restriction Choking at J2 will become unchoked and no choking will occur anywhere in the system

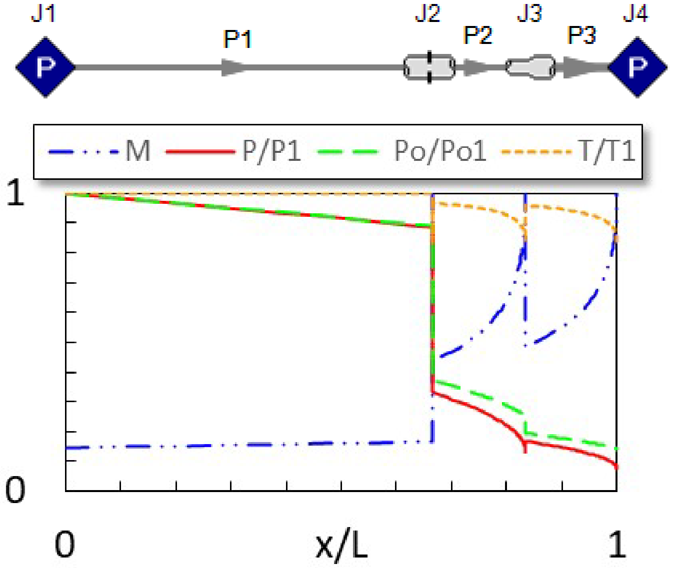


Input	English Units	Metic Units
Pipes 1 & 2, Jcts 1 & 2 - see Fig. 5		
Pressure (Static) Outlet (J4)	50 psia	344.7 kPa
Output (Pipe 1)		
Same as Example 6.3, Pipe 1		
Output (J2 at Min. Flow Area)		
Same as Example 6.3, Junction 2		
Output (Pipe 2)		
Mach number Inlet	0.439	0.439
Pressure (Stag.) Inlet	148.6 psia	1024.9 kPa
Pressure (Static) Inlet	130.2 psia	897.7 kPa
Temperature (Static) Inlet	175.5 F	79.7 C
Velocity Inlet	542.7 ft/s	165.5 m/s
Mach number Outlet	1.0	1.0
Pressure (Stag.) Outlet	100.7 psia	694.4 kPa
Pressure (Static) Outlet	53.2 psia	366.8 kPa
Temperature (Static) Outlet	90.1 F	32.3 C
Velocity Outlet	1149.4 ft/s	350.4 m/s

FIGURE 10: EXAMPLE 7.2 OF RESTRICTION CHOKING WITH TWO CHOKING POINTS IN SERIES

7.4 General Discussion of Multiple Choking Points in Series

When considering a sequence of connected pipes, there is no limit to the number of sonic choking points – other than the number of geometric locations which permit sonic choking (e.g., the number of pipe restrictions and expansions, including Section 3.4 exceptions). There can be only one Endpoint Choking for a pipe sequence.



Input	English Units	Metic Units
Pipes 1-3, Jcts 1 & 2 - see Fig. 5		
Pressure (Static) Outlet (J4)	25 psia	172.4 kPa
Output (Pipe 1)		
Same as Example 6.3, Pipe 1		
Output (J2 at Min. Flow Area)		
Same as Example 6.3, Junction 2		
Output (Pipe 2)		
Same as Example 7.2, Pipe 2		
Output (Pipe 3)		
Mach number Inlet	0.485	0.485
Pressure (Stag.) Inlet	77.6 psia	535.1 kPa
Pressure (Static) Inlet	66.1 psia	455.6 kPa
Temperature (Static) Inlet	170.4 F	76.9 C
Velocity Inlet	596.6 ft/s	181.9 m/s
Mach number Outlet	1.0	1.0
Pressure (Stag.) Outlet	56.7 psia	390.6 kPa
Pressure (Static) Outlet	29.9 psia	206.3 kPa
Temperature (Static) Outlet	90.1 F	32.3 C
Velocity Outlet	1149.4 ft/s	350.4 m/s

FIGURE 11: EXAMPLE 7.3 WITH THREE CHOKING POINTS IN A SERIES OF CONNECTED PIPES

A reduction in pipe diameter cannot support a sonic choking point under adiabatic conditions (with Section 3.4 exceptions) and the vast majority of non-adiabatic flow conditions (see Footnote 1 in Section 3.2). For example, if the pipes in Fig. 9 were reversed and Pipe P1 was the larger diameter, sonic choking cannot occur at the J3 contraction under most normal conditions. If sonic choking occurred, it would only be at the outlet of Pipe P3 as Endpoint Choking.

If a reduction in pipe diameter cannot support sonic choking, how can Restriction Choking occur? Isn't that a reduction in diameter? Yes, of course it is. But the difference with Restriction Choking is that after the restriction the diameter increases again. This allows the choking to occur at the orifice. With a reduction in pipe diameter, there is no subsequent increase in pipe diameter – just the sustained reduction in diameter.

In general, a sequence of connected pipes can have a mix of choked and unchoked pipes and junctions. The only practical way to solve this is to use the approaches described in this paper starting from the inlet of the entire pipe sequence and making your way down each pipe to the outlet of the sequence. When adding in more realistic situations such as real gas behavior, heat transfer, elevation changes, and variable friction, capable software solutions *designed specifically to handle all these cases* become the only practical way of solving this.

8. MULTIPLE CHOKING POINTS IN PARALLEL FLOW PATHS

8.1 Four Choking Points in Parallel Flow Paths (Example 8.1)

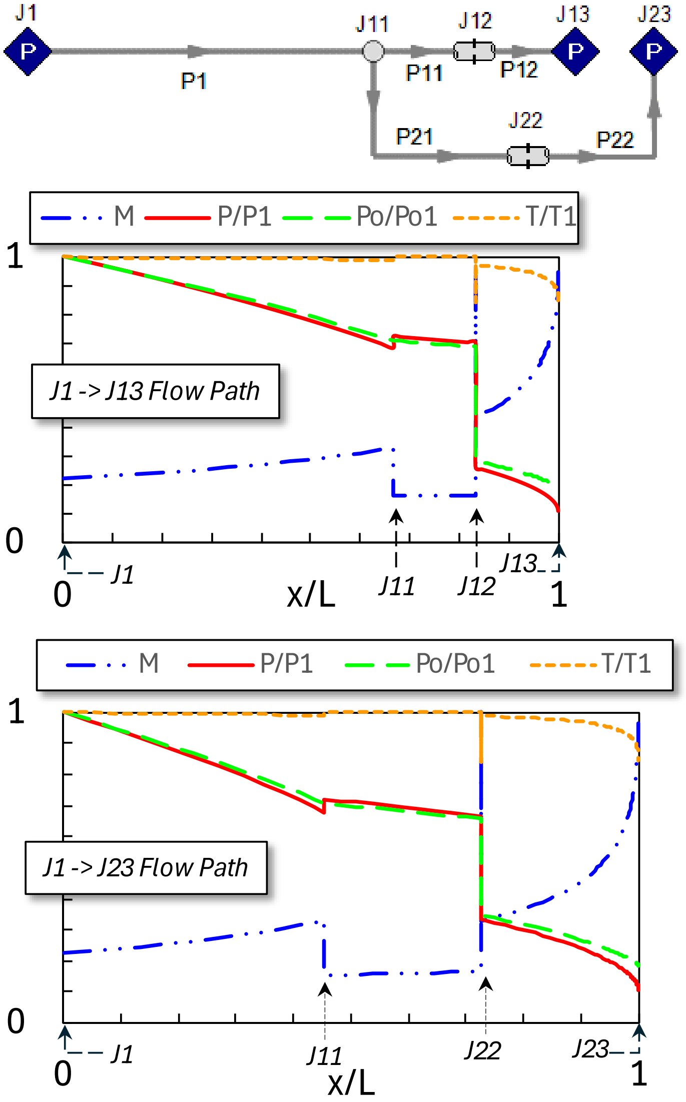
Fig. 12 shows a system with a single supply and two discharges in parallel. The branch node at J11 is assumed to be lossless for simplicity. The conditions for this system result in four sonic choking points:

- Two choking points at the J12 and J22 orifices which experience Restriction Choking
    - Each of these controls the flowrate for the respective downstream pipes
- Two choking points at the J13 and J23 outlet tanks which experience Endpoint Choking at the outlet of Pipes P12 and P22

As seen in Fig. 12, the system is not symmetric with Pipes P21-22 being longer than Pipe P11-12. The Appendix shows all output results in numerical form.

First, the flow in the two parallel paths is different because of the asymmetry. Second, the Endpoint Choking static pressures in Pipes P12 and P22 are different – 41.4 psia (286 kPa) in Pipe P12 and 39.8 psia (274 kPa) in Pipe P22. Interestingly, it would be possible to find a common pressure in J13 and J23 which would result in Endpoint Choking in one path but not the other. This would be any static pressure between 39.8 and 41.4 psia (274 and 286 kPa). Third, predicting the sonic choking at the J12 and J22 orifices is quite challenging. One must use Eq. 5 which governs the flowrate through J12 and J22. However, the choked flow through each orifice requires a solution for the stagnation pressure and temperature *at each orifice*. That requires solving the conditions in Pipe P1, determining the conditions at the J11 branch, and then solving Pipes P11 and P21 up to each orifice. The stagnation pressures at J12 and J22 will be different because of the asymmetry (see the Appendix).

Another item of interest is that there is another possible location of sonic choking in this system. If the total discharge area downstream of the J11 branch is greater than that of Pipe P1, it is possible for Pipe P1 to experience Expansion Choking at its exit into J11. Is this possible here? In fact, it is. The area of Pipe P1 is 7.1 in^2 (45.6 cm^2). The area of each Pipe P11 and P21 is also 7.1 in^2 (45.6 cm^2). The two pipes together are thus 14.1 in^2 (91.2 cm^2) and thus greater than Pipe P1. Hence, Expansion Choking is possible. However, the Appendix results show an outlet Mach number of Pipe P1 of 0.33, which is far below the choking point. Thus, there is no Expansion Choking at J11.



Input	English Units	Metic Units
Pipe 1, Jct 1, 12, 22 - see Fig. 5		
Pressure (Static) Outlet (J13, J23)	35 psia	241.3 kPa
Diameter (P11-12, P21-22)	3 in.	7.62 cm
Length (P11-12)	25 ft	7.6 m
Length (P21-22)	60 ft	18.3 m
Friction Factor (P11-12, P21-23)	0.017	0.017

FIGURE 12: EXAMPLE 8.1 WITH MULTIPLE CHOKING POINTS IN PARALLEL FLOW PATHS

Predicting the performance of the Fig. 12 system is complicated even for the idealized assumptions in this paper. Therefore, a solution was obtained with a commercially available software package [6].

9. GENERAL RECOMMENDATIONS ON ANALYZING PIPE NETWORK SYSTEMS WITH SONIC CHOKING

Even for the relatively simple system in Example 8.1, a software solution is the only practical approach. Below is a summary of the many technical issues covered in this paper that must be considered to analyze a general gas pipe network. An engineer evaluating software to assist with this task should ensure the software can handle these issues.

Required Minimum Capabilities

1. All types of sonic choking geometries discussed in Section 3
2. Ability to predict conditions downstream of choke points as discussed in Section 6.4
    a. Note this will require a proper solution of the energy equation across each choke point (constant stagnation enthalpy)
3. Ability to predict multiple choke points in series and in parallel
    a. Note that obtaining a valid converged solution for parallel path choke points can be quite challenging and engineers should verify this (at a minimum, the software should be able to accurately solve the relatively simple Example 8.1)
4. Ability to predict flows, pressure drops and thermal conditions from the supply point up to each choke point so that Eq. 5 can be solved with valid solutions at the choke point (and not some point further upstream)

Desirable Capabilities

5. Handle real gas modeling (even for air, especially at higher pressure and/or temperature conditions)
6. Handle heat transfer modeling to ambient conditions
7. Ability to handle pipe elevation changes

10. CONCLUSIONS

Sonic choking can occur in multiple places in a general pipe network system. Understanding where and why the choke points exist can be crucial in understanding the overall system performance and assessing safety. This paper describes all the possible sonic choking cases and provides an understanding of how to predict each case. Cautions are provided on when closed-form analytic solutions break down and when to look for more sophisticated software solutions.

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APPENDIX – NUMERICAL RESULTS FOR EXAMPLE 8.1

	Output	**Units**	**P1**		**P11**		**J12 @MIN. AREA**		**P12**		**P21**		**J22 @MIN. AREA**		**P22**	
	Mass flow rate	lbm/s | kg/s	22.2	10.1	11.3	5.1	11.3	5.1	11.3	5.1	10.9	4.9	10.9	4.9	10.9	4.9
Inlet	Mach number	-	0.226	0.226	0.161	0.161			0.439	0.439	0.154	0.154			0.330	0.330
	Pressure (Stag.)	psia | kPa	400.0	2757.9	282.7	1949.1			114.3	788.1	282.7	1949.1			139.0	958.4
	Pressure (Static)	psia | kPa	386.0	2661.4	277.7	1914.7			100.1	690.2	278.1	1917.4			128.9	888.7
	Temperature (Static)	deg F | C	193.3	89.6	196.6	91.4			175.6	79.8	196.9	91.6			186.0	85.6
	Velocity	ft/s | m/s	283.4	86.4	202.1	61.6			542.7	165.5	193.7	59.1			411.0	125.3
Outlet	Mach number	-	0.331	0.331	0.166	0.166	1.0	1.0	1.0	1.0	0.166	0.166	1.0	1.0	1.0	1.0
	Pressure (Stag.)	psia | kPa	282.7	1949.4	273.9	1888.3	273.9	1888.3	77.4	533.9	262.8	1811.8	262.8	1811.8	74.3	512.3
	Pressure (Static)	psia | kPa	262.0	1806.6	268.6	1852.1	144.7	997.7	41.4	285.7	257.8	1777.1	138.8	957.0	39.8	274.3
	Temperature (Static)	deg F | C	185.9	85.5	196.4	91.3	90.0	32.2	92.3	33.5	196.4	91.3	90.0	32.2	92.4	33.5
	Velocity	ft/s | m/s	412.8	125.9	208.8	63.7	1,149.0	350.3	1138.8	347.2	208.8	63.7	1,149.0	350.3	1138.3	347.0