

## PUMP SPECIFIC SPEED AND FOUR QUADRANT DATA IN WATERHAMMER SIMULATION – TAKING ANOTHER LOOK

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### ABSTRACT

*In some situations, it is possible for flow to go backwards through a pump during a transient waterhammer event. Sustained reverse flow will lead to reverse rotation. Understanding and predicting the pump behavior during waterhammer under these conditions is typically accomplished using previously published four quadrant pump data. Historically, the selection of which data to use is based on the similarity of pump specific speed. The weaknesses of using specific speed are described and an improved method of selecting appropriate four quadrant data is given based on fundamental curve shapes for head and power in the normal operating zone.*

Keywords: Waterhammer, four quadrant pump, Suter Method, Stepanoff

### NOMENCLATURE

BEP	Best Efficiency Point (%)
Ns	Specific Speed of Pump (dimensionless, but by tradition made dimensional in US units as RPM-USgpm <sup>0.5</sup> / ft <sup>0.75</sup> and in Metric units as RPM-(m <sup>3</sup> /s) <sup>0.5</sup> / m <sup>0.75</sup> )

### Suter dimensionless parameters for four quadrant pump representation [1], [2]

$h$	dimensionless head (pump operating head divided by the rated head)
$\alpha$	dimensionless speed (pump operating speed divided by the rated speed)
$\beta$	dimensionless torque (pump operating torque divided by the rated torque)
$\nu$	dimensionless flow (pump operating flow divided by the rated flow)
$FH$	$h / (\alpha^2 + \nu^2)$
$FB$	$\beta / (\alpha^2 + \nu^2)$
$\theta$	$\tan^{-1} (\alpha / \nu)$

### 1. INTRODUCTION

The possibility of reverse pump flow or rotation during a waterhammer event requires the use of methods for estimating pump behavior in all four quadrants. For the last five decades the state-of-the-art has been the Suter Method (Marchal, Flesch and Suter [1], and Suter [2]). This method correlates pump data sets into dimensionless form. Except in the rare cases when engineers have four quadrant data for their pump, the typical modeling choice is to use data for a previously tested pump with a similar specific speed. Some publications over the years have questioned the correctness of using pump specific speed as a correlating factor. This paper explores this concern.

First, in order to correlate four quadrant pump data based on specific speed, true dimensional and dynamic similarity between pumps must exist. Modern insights of pump performance and pump specific speed will be given to assess the effectiveness of the assumption of correlation.

Second, some of the excellent four quadrant data used today was first published as far back as the 1930's – see Knapp [3] and Swanson [4]. The researchers could not have envisioned how their data would be used in modern waterhammer simulation. Indeed, many of the data sets used today were taken before Suter published his method in the 1960's and before digital computers were created. As a result, it is not clear that the researchers placed a strong emphasis on appropriately calculating and reporting the specific speed of the pumps used in their testing. This paper will attempt to confirm or disconfirm whether the correct specific speeds were reported.

Finally, the Suter Method is a convenient and creative dimensionless formulation of pump data referenced to the Best Efficiency Point (BEP) of the test pump. However, researchers over the years have taken to only publishing the dimensionless data and not the dimensional data behind the test. This makes it impossible to compare the waterhammer analyst's pump to the original pump's dimensional data, especially when the analyst's

pump is not operating near its own BEP – which is often the case. See Walters, Lang and Miller [5] for a comprehensive summary of the history and evolution of four quadrant pump research in the literature, a good overview of the Suter Method (in Part 1 of their paper), and a thorough explanation of the implications of off-BEP operation.

These issues are explored and recommendations are given to improve how pumps are modeled in waterhammer simulation.

## 2. REVIEW OF PUMP FOUR QUADRANT METHODS AND USE OF SPECIFIC SPEED

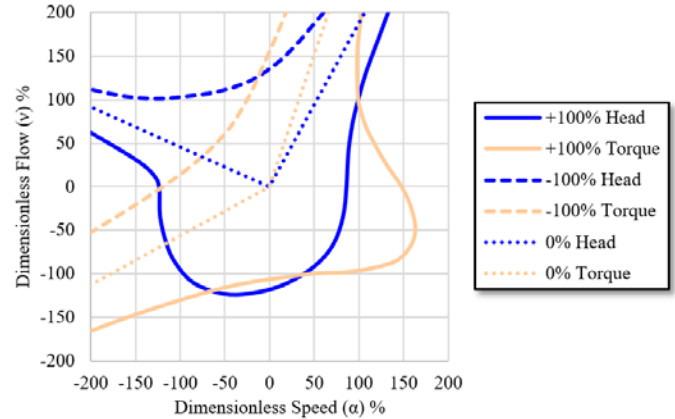
There are two pump system configurations that are especially susceptible to reverse flow and rotation after a pump trip:

- A rising main, where gravity will pull the discharge fluid back towards the pump
- Parallel pump operation where one pump trips and the others remain in operation

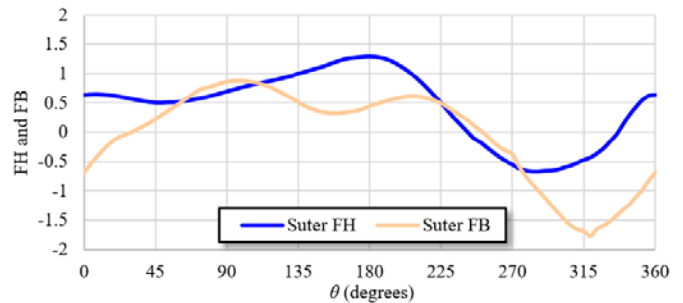
Many pump systems have check valves to prevent reverse flow. Such configurations have their own waterhammer issues, which is well documented in the literature. For example, see Lozano, Bosch and Walters [6]. But there are some pump systems where check valves are not or cannot be used. These include slurry systems where the forward flow of solid material can damage or clog check valves, and large diameter piping such as that used on major cooling water systems such as those found on power plant condenser systems.

When such systems experience either a planned or unplanned pump trip, engineers need to be able to predict the waterhammer caused by the pump trip even when it is flowing or rotating in reverse. Data for pumps that operate in this manner are commonly called four-quadrant data. The three most popular sources of four quadrant data were generated at CalTech in the first half of the 20th century by Knapp [3] for radial flow pumps and Swanson [4] for mixed flow and axial flow pumps. Later Donsky [7] took the Knapp and Swanson data and, using pump affinity laws in all four quadrants, painstakingly created detailed dimensionless four quadrant pump charts for radial flow, mixed flow and axial flow pumps. Knapp, Swanson and Donsky all commented on how, for the most part, the data shows that pumps follow affinity laws even when not operating in the normal positive rotation, flow and head manner.

The Suter method (Marchal, Flesch and Suter [1], and Suter [2]) provides a convenient way to aggregate the four-quadrant data into dimensionless form. When engineers need to predict four quadrant pump behavior, and they do not have such data for their pump, which is typical, the state-of-the-art recommends they look in the literature for a previously published set of data for a pump with a similar specific speed. Fig. 1 shows four quadrant data obtained by Knapp [3] as presented by Donsky [7] for a radial flow pump at 100%, -100% and 0% of rated conditions as a function of dimensionless speed and flowrate. Fig. 2 shows the Suter curves of the data. It is not the intent of this paper to provide a thorough review of the Suter Method. See Walters, Lang and Miller [5] for a detailed summary.



**FIGURE 1: FOUR QUADRANT TEST DATA FROM A RADIAL FLOW PUMP (KNAPP [3]),  $N_s = 1270$  US/24.6 METRIC. USED WITH PERMISSION FROM WALTERS, LANG AND MILLER [5].**



**FIGURE 2: SUTER CURVES CONSTRUCTED FROM A RADIAL FLOW PUMP. THE SPECIFIC SPEED FOR THE TEST PUMP IS  $N_s = 1270$  US/24.6 METRIC. USED WITH PERMISSION FROM WALTERS, LANG AND MILLER [5].**

## 3. USING PUMP SPECIFIC SPEED AS A CORRELATING FACTOR

Many previous studies have questioned the use of pump specific speed as an accurate correlating factor when selecting previously published four quadrant pump data. Brown and Rogers [8] especially focused on this issue for radial flow pumps. They suggested the issue was not as significant for mixed flow and axial flow pumps.

There are three parts to this issue:

1. Whether the specific speed of the tested pump was reported correctly
2. Whether a tested pump's four quadrant curve is a good representation of all pumps at that specific speed
3. Whether use of specific speed as the sole correlating factor is valid in general

These issues will be explored in the following sections.

### 3.1 Investigation of Pump Specific Speed Reporting Accuracy in Published Four Quadrant Data

Walters, Lang and Miller [5] point out that the calculation of specific speed in accordance with published standards can be confusing. Unfortunately, it is typical in the past and still today for authors to only publish the dimensionless Suter data and not the dimensional data for the test. Indeed, it is rare to see published in the waterhammer literature the rated conditions for the pump, its size or its impeller diameter, much less its dimensional performance data (the pump head and power curves) in normal operating mode. Since the three most referenced and trusted data sets were obtained from research at CalTech (Knapp [3], and Swanson [4]) the first author went to CalTech in June 2018 and searched through its library archives for any pump dimensional data pertaining to these tests. This involved looking through many boxes of unpublished loose-leaf paper and photos. Unfortunately, this was not successful.

As a next step the manufacturers of the original pumps were consulted to see if they had any record of the CalTech pumps in their archived data. This was Flowserve for the Knapp pump and Grundfos for the Swanson pumps. This also was not successful. It is reasonable to conclude that the data has been lost to history and we will never know.

The first author has not had any success in identifying the underlying pump data for any of the other commonly used four quadrant pump data sets. This includes 6 four quadrant data sets reported by the third author of this paper (Brown and Rogers, [8]), which unfortunately were discarded in subsequent years.

For any tests done on single stage pumps with the maximum impeller diameter, it is likely the specific speed was reported properly when the four quadrant data was published (trimmed impellers and multiple stages are the most likely causes of errors in calculation of specific speed). If the pumps had a trimmed impeller, and the authors incorrectly used the flow rate and head for the trimmed impeller pump they used, rather than the maximum impeller for that pump, the reported specific speeds would be too high. If the pump had multiple stages and the authors did not account for that, it is likely the reported specific speed would be too low. Lastly, the total flowrate in double suction impellers is divided in half between the two impeller eyes. The normal convention for the calculation of pump specific speed is to use the total pump flow (see ANSI-HI 1-1-1.2-2014, Paragraph 1.1.4.1. [9]). An alternative definition for specific speed is sometimes based on the flow per impeller eye. This reduces the calculated pump specific speed by a factor of  $1/\sqrt{2}=0.707$  for double suction impellers. This alternative method is another source for differences in the calculated value of reported pump specific speed.

Recommendations for future four quadrant test programs are given later in this paper.

### 3.2 Investigation of Published Four Quadrant Data Providing a Good Representation of All Pumps at the Same Specific Speed

The Pump Handbook (Karassik et al., [10]) provides insightful data first published by Stepanoff [11] on the shape of pump curves for normal pump operation in the first quadrant. These pump flow versus head and flow versus power performance curves are normalized with respect to the BEP values at various specific speeds. These dimensionless performance curves are the dashed curves for head and power vs. flow rate in Figs. 3 and 4. These Stepanoff curves depict well known trends for the general shape of head and power characteristics as a function of specific speed. There is a statistical dispersion of pump data that does not conform identically to these dimensionless Stepanoff curves, but they are good indicators of pump specific speed based on general curve shape and slope.

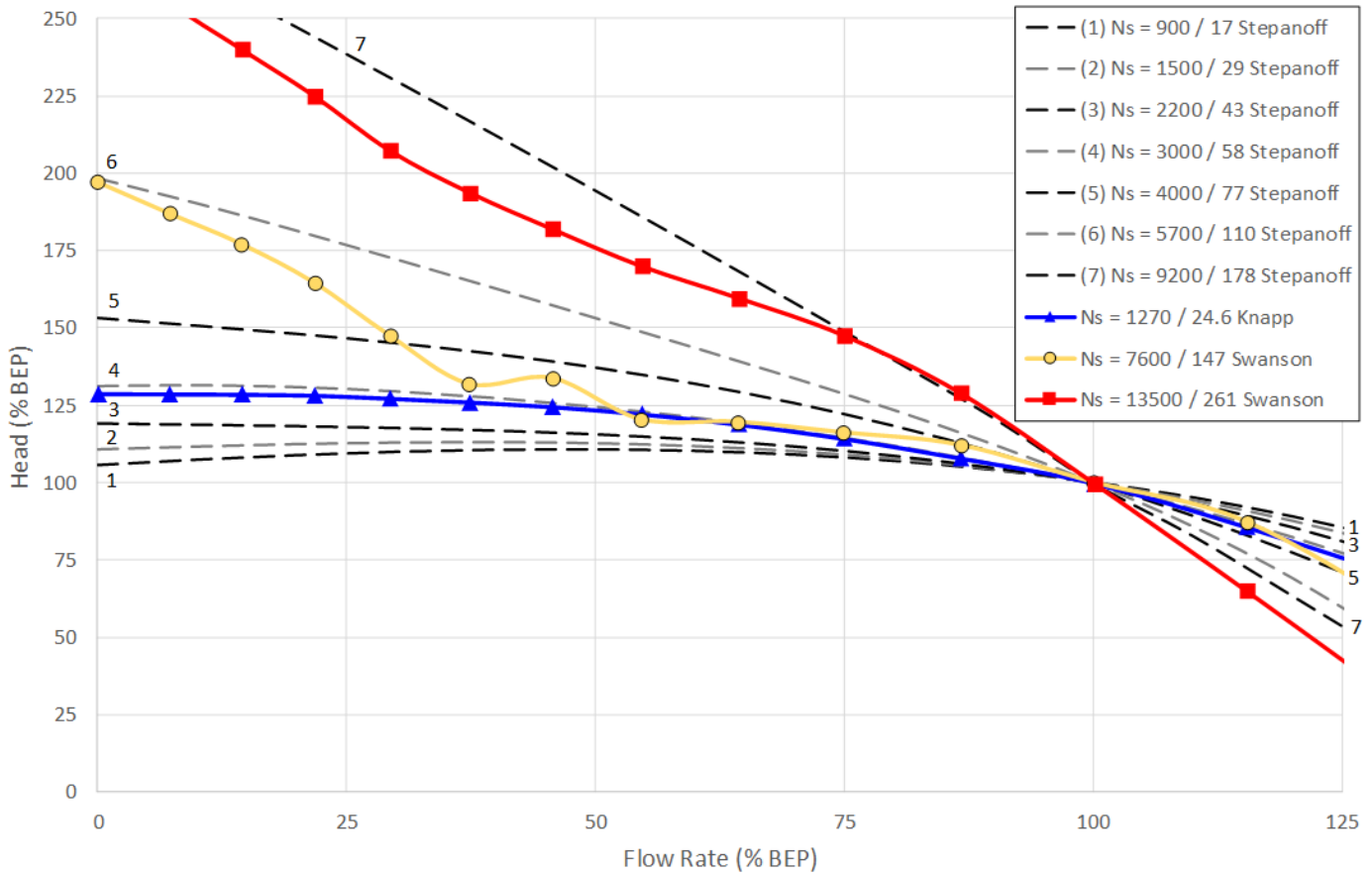
Figs. 3 and 4 also show the three CalTech curves (Knapp [3], and Swanson [4]) which have been semi-dimensionalized from their typical Suter form in the normal pump operation zone.

One striking thing apparent in Figs. 3 and 4 is that the CalTech curves do not follow the Stepanoff shapes very well at all, especially the two Swanson curves. The Knapp curve roughly follows the shapes for Stepanoff at the specific speed of 2,200 US / 43 Metric. This pump is a double suction pump, and Knapp's data was reported by Knapp and also by Swanson as specific speed 1,800 US / 35 Metric. Subsequent publications (e.g., Brown and Rogers [8] and virtually all modern textbooks) modified this to 1,270 US / 25 Metric by dividing the flow in half because it was double suction. However, Figs. 3 and 4 provide strong evidence this was done in error and Knapp and Swanson (and also, later, by Donsky, [7]) were correct in their original report of specific speed. It is recommended that the Knapp data Suter curve be categorized as 1,800 US / 35 Metric in accordance with the more conventional specific speed calculation. This is also in agreement with how ANSI-HI 1-1-1.2-2014 [9] defines specific speed, as noted earlier.

From Donsky ([7], Fig. 9) it is evident that the 7600 US / 147 Metric curve from Swanson had an instability at roughly 60% of BEP. This can occur in vertical turbine pumps which can also explain the odd shape of the curve and lack of agreement with Stepanoff in Fig. 3.

The second Swanson curve for specific speed 13,500 US / 261 Metric follows Stepanoff much better. However, it follows the Stepanoff 9200 US / 178 Metric curve better for head and the 5700 US / 110 Metric curve for power. This could be a case of misreported specific speed by Swanson. This will be discussed further in Section 5 of this paper.

As noted previously, the confusion here with all three CalTech curves could be resolved if the original pump data sheets had been retained. Since they have not, there is no way to know what happened in these tests. One would hope that Knapp and Swanson had checked their data against the data sheet for the pump. If it did not agree, that would call something into question about their test.



**FIGURE 3: STEPANOFF HEAD CURVES FROM KARASSIK ET AL. [10] AT VARIOUS SPECIFIC SPEEDS (SHOWN IN US / METRIC), ALONG WITH THE THREE MOST COMMON FOUR QUADRANT PUMP DATA CURVES FROM KNAPP [3] AND SWANSON [4] IN THE NORMAL PUMP OPERATING ZONE.**

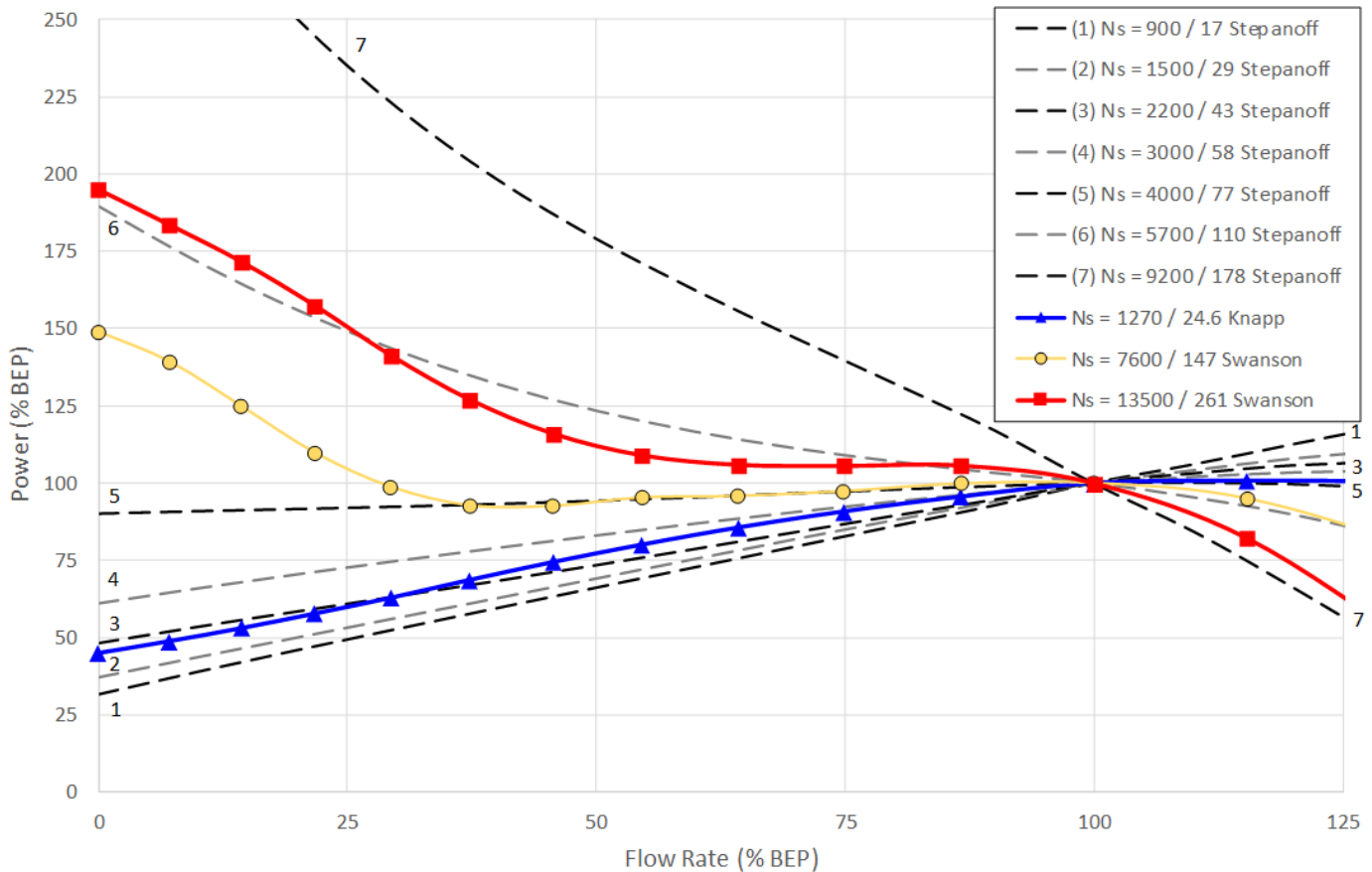
The Stepanoff data and experience of the second author is that pump curves tend to follow well established shapes and changes in slope in the zone of normal operation as a function of specific speed. It stands to reason that the same would be true in all four quadrants based on the similarity in pump performance that is found with specific speed. It also stands to reason that if a reported four quadrant data set does not follow trends in the Stepanoff curve in the zone of normal operation (the easiest zone to verify for the testers), it likely provides poor data in all four quadrants.

Addressing more directly the titled topic of this section, the authors support the belief that published four quadrant data is a good representation for all pumps of similar specific speed under these two conditions:

1. The reported specific speed is correct
2. The dimensionalized data in the zone of normal pump operation agrees with Stepanoff

### 3.3 Investigating the Use of Specific Speed as the Sole Correlating Factor for Four Quadrant Data

The previous discussion using Stepanoff supports the belief that specific speed is appropriate for use as the sole correlating factor for four quadrant pump data – when the data is determined to be valid at that specific speed. A recommendation on this point will be made in a later section.



**FIGURE 4: STEPANOFF POWER CURVES FROM KARASSIK ET AL. [10] AT VARIOUS SPECIFIC SPEEDS (SHOWN IN US / METRIC), ALONG WITH THE THREE MOST COMMON FOUR QUADRANT PUMP DATA CURVES FROM KNAPP [3] AND SWANSON [4] IN THE NORMAL PUMP OPERATING ZONE.**

#### 4. ISSUES AND RECOMMENDATIONS WHEN NOT OPERATING AT BEST EFFICIENCY POINT

Walters, Lang and Miller [5] provide a thorough review of the potential impacts of using four quadrant pump estimation methods in off-BEP operation for radial flow pumps. In summary, they pointed out that in off-BEP operation a mismatch in steady-state pump operation is usually introduced which can have a significant impact on the waterhammer transient results. In that a large majority of industrial pumping systems are not operated at BEP, this issue impacts almost all pumping system waterhammer analyses.

During the presentation of the Walters, Lang and Miller papers it was pointed out by the attendees that a hybrid “splice” method could be used similar to that of Wan and Huang [12] to avoid the mismatch between a pump manufacturer’s curve and the dimensionless four quadrant data when operating in off-BEP operation. The so-called “splice method” is an arbitrary alteration of the measured data behind a four-quadrant pump curve and its impact on waterhammer simulation needs to be investigated. Further, a study of off-BEP operation for mixed flow and axial flow pumps is still needed similar to that of Walters, Lang and Miller for radial flow pumps.

For pumping system waterhammer analysts there are often multiple objectives among which are to:

- Identify the maximum and minimum transient pressures for pipe pressure rating and wall thickness adequacy
- Identify the maximum unbalanced loads for pipe structural support design
- In some cases, size surge suppression systems such as relief valves, vacuum breaker valves and surge vessels

Walters, Lang and Miller [5] recommended that until better guidance from waterhammer researchers can be provided on off-BEP operation, waterhammer analysts should broaden the scope of sensitivity cases they evaluate to include the various assumptions on pump steady-state off-BEP operation. In addition, waterhammer analysts should include in their studies multiple published four-quadrant data sets with a similar specific speed to their pump.

## 5. REVIEW OF COMMONLY CITED PUMP FOUR QUADRANT DATA

Of note in Fig. 1 is that the data acquired by Knapp was taken in steady-state operation of the pump in all four quadrants. However, waterhammer simulation assumes this data also applies accurately to the transient pump behavior. This is a largely untested assumption. One study that attempted to evaluate this assumption is Gros et al. [13]. Gros et al. observed a difference in transient vs. steady-state behavior in Quadrants 2 and 4 (i.e., in Fig. 1, the upper left and lower right quadrants).

It is difficult to generalize time scales for waterhammer simulation. On some small-scale systems, the important time scales occur in fractions of a second. On larger pipeline systems, the time scale can be in minutes or 10's of minutes. The time scale also can impact the steady-state assumption.

Table 1 shows a review of the quality of published four quadrant data in the zone of normal pump operation as compared to Stepanoff [11] and Karassik et al. [10]. Charts for all data sets

are not shown for space reasons but are available as a downloadable Excel file (see Walters, Dahl and Rogers, auxiliary data files. [14]). Of the 26 data sets in Table 1, it is recommended two (items 3 and 5) be recategorized to a specific speed reported by the original authors. It is recommended that eight data sets not be used by analysts. It is further recommended that five of the data sets be used with caution. Thirteen data sets are recommended for unconditional use.

In assessing curves 24, 25 and 26 in Table 1 (see Walters, Dahl and Rogers [14]), it became apparent that all three curves differed from the Stepanoff curve 7 (in Figs. 3-4) in a consistent manner. This led to closer inspection and it became evident that Stepanoff's curve 7 has an issue with head or power or both, especially to the right of the BEP (i.e., flow 100-125% of BEP). Specifically, curve 7 does not yield a maximum efficiency at the claimed BEP. Higher efficiencies are obtained to the right of what Stepanoff claims is the BEP. This is not possible. This Stepanoff curve 7 should be viewed by all as possibly unreliable.

**TABLE 1: REVIEW OF QUALITY OF PUBLISHED FOUR QUADRANT DATA WHEN COMPARED TO STEPANOFF CURVES AND OVERALL CONSISTENCY. SPECIFIC SPEED IN US / METRIC UNITS**

4Q Data	Reported Specific Speed		Head Comparison	Power Comparison	Overall Comparison	Recommendation	Reference
	US	Metric					
1	810	15.7	Good	Fair	Fair	Use as is	[8] Brown and Rogers 1980
2	1030	20.0	Fair	Very Good	Good	Use as is	[15] Ayder et al. 2009
3	1060	20.5	Fair	Good	Fair	Double-suction, originally reported as $N_s = 1500 / 29$ . Data matches the higher $N_s$ better. Recommend for use at $N_s = 1500 / 29$ .	[16] Kittredge 1956
3*	1500	29.1	Good	Very Good	Good	Kittredge originally reported $N_s$ of $1500 / 29$ .	[16] Kittredge 1956
4	1140	22.1	Fair	Very Good	Good	Use as is	[8] Brown and Rogers 1980
5	1270	24.6	Fair	Fair	Fair	Double-suction, originally reported as $N_s = 1800 / 35$ . Data matches the higher $N_s$ better. Recommend for use at $N_s = 1800 / 35$ .	[3-4] Knapp 1937 / Swanson 1953
5*	1800	34.9	Good	Very good	Good	Knapp originally reported $N_s$ of $1800 / 35$ .	[3-4] Knapp 1937 / Swanson 1953
6	1320	25.6	Very poor	Poor	Very poor	Do not use	[8] Brown and Rogers 1980
7	1490	28.9	Poor	Very poor	Very poor	Do not use	[8] Brown and Rogers 1980
8	1565	30.3	Fair	Very poor	Poor	Do not use	[8] Brown and Rogers 1980
9	1700	33.0	Very good	Very good	Very good	Use as is	[15] Ayder et al. 2009
10	1935	37.5	Very good	Very good	Very good	Use as is	[16] Kittredge 1956
11	2160	41.9	Poor	Very Good	Fair	Use with caution	[17] Thorley 1996
12	3300	64.0	Poor	Good	Fair	Use with caution	[17] Thorley 1996
13	3725	72.2	Good	Very good	Good	Use as is	[17] Thorley 1996
14	3940	76.4	Very good	Very good	Very good	Use as is	[17] Thorley 1996
15	4400	85.3	Very good	Very good	Very good	Use as is	[8] Brown and Rogers 1980
16	5000	96.9	Very good	Fair	Good	Use as is	[17] Thorley 1996
17	5200	100.8	Fair	Poor	Poor	Do not use. Data also shows signs of cavitation.	[17] Thorley 1996
18	5420	105.0	Very good	Poor	Fair	Use with caution.	[15] Ayder et al. 2009
19	6340	122.9	Fair	Poor	Poor	Do not use	[17] Thorley 1996
20	6790	131.6	Poor	Poor	Poor	Do not use. Data also shows signs of cavitation.	[17] Thorley 1996
21	6940	134.5	Poor	Very poor	Very poor	Do not use. Data also shows signs of cavitation.	[17] Thorley 1996
22	7600	147.3	Fair	Fair	Fair	Data does not follow Stepanoff but is similar to some vertical pumps with instabilities. Use with caution.	[14] Swanson 1953
23	8760	169.8	Poor	Poor	Poor	Do not use. Data also shows signs of cavitation.	[17] Thorley 1996
24	10800	209.0	Good	Fair	Fair	Use with caution	[15] Ayder et al. 2009
25	13500	261.6	Good	Fair	Good	Use as is	[14] Swanson 1953
26	13500	261.6	Good	Fair	Good	Use as is	[15] Ayder et al. 2009

## 6. RECOMMENDATIONS

### 6.1 Future Pump Four Quadrant Test Program Recommendations

1. Include the pump data sheet with the publication
2. Identify all important data including:
  - a. impeller size
  - b. maximum impeller size
  - c. # stages
  - d. BEP conditions for head, flow and power for design impeller size
  - e. BEP head, flow and power at maximum impeller size
  - f. RPM
3. Check normal pump operation data against manufacturer data sheet and Stepanoff curves and report anomalies

### 6.2 Waterhammer Analysis Recommendations

1. Discontinue choosing pump four quadrant data solely on the basis of specific speed
2. Consult Table 1 for a review of four quadrant data quality and choose higher quality data sets when possible
3. Compare shape of head and power curves for pump to be analyzed vs. published four quadrant curves in dimensionalized normal operating zone and choose four quadrant data sets which agree best with pump to be analyzed
4. Pay attention to how close to BEP the pump operates and the agreement on predicted initial steady-state flow rates between the pump to be analyzed and the four quadrant pump curve – choose four quadrant data sets which provide better agreement to initial operating point
5. Pay attention to how close to BEP the pump operates and the agreement on predicted final steady-state flow rates between the pump to be analyzed and the four quadrant pump curve – choose four quadrant data sets which provide better agreement to final operating point (after all transients have died out)
6. As suggested by Walters, Lang and Miller [5], expand sensitivity studies to evaluate impacts from different four quadrant curves and effects from off-BEP operation on initial, steady-state flow rates

Items #4 and 5 are discussed at length in Walters, Lang and Miller [5].

## 7. CONCLUSIONS

Four quadrant pump data available in Suter form has been the state-of-the-art for over five decades. Correlation of data based on pump specific speed has been the typical approach for selecting four quadrant data. An improved selection method is

described based on comparisons to the shape of head and power curves in the zone of normal operation and comparisons to standard curve shapes from Stepanoff [11].

Highly referenced data from Knapp [3] and Donsky [7] was originally reported as specific speed of 1,800 US / 35 Metric. Because the pump was a double-suction pump, their data was later claimed to be valid for a specific speed of 1270 US / 25 Metric and has been used for that specific speed for roughly the last 40 years by virtually all waterhammer analysts. The analysis here indicates that the Knapp data should be recategorized back to 1,800 US / 35 Metric. A similar situation applies to lesser known data by Kittredge [16] which he originally reported as specific speed 1500 US / 29 Metric.

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