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**PULSATION ANALYSIS IN POSITIVE DISPLACEMENT PUMP SYSTEMS USING WATERHAMMER, MODAL AND ANIMATION SOFTWARE**

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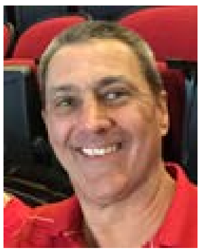

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## ABSTRACT

High-pressure and flow metering systems often include a positive displacement (PD) pump and a network of piping and process equipment. Steady state pressure and flow dynamics (pulsation) is a common problem in PD pump systems, which can cause high vibration, fatigue failures, frequent maintenance outages, and flow uniformity or product quality problems.

A number of engineering consulting firms have developed computational capabilities in pulsation, which they offer in the form of client services, but these are expensive. So too is the limited amount of commercial pulsation-specific software.

This paper describes a methodology to computer model pulsation using a combination of general-purpose and readily available, widely used software.

*Waterhammer Software* – Normally used to simulate transients, if periodic flow forcing is modeled, the transients die out, and leave only the steady-state pulsation

*Modal Software* – Frequency analysis of high-speed digital time history data can be done with many software platforms, including those for Experimental Modal Analysis (EMA), Data Acquisition, or general-purpose Statistical/Mathematical modeling

*Animation Software* – Typically included with EMA software, animation is not necessary, but is useful to visually show pressure and flow response across the system at acoustic natural frequencies and for PD pump flow forcing

An example problem shows how to efficiently model complex systems to determine acoustic natural frequencies, mode shapes, and pulsation response to PD pump forcing. Fundamentals of waterhammer and important modeling techniques are discussed. An introduction to EMA and digital signal processing is also provided.

## INTRODUCTION

High-pressure and flow metering systems often include a positive displacement (PD) pump and a network of piping and process equipment components. An example is shown in Fig 1 of such a suction system.

Steady state pressure and flow dynamics, henceforth referred to simply as "pulsation", is a common problem in PD pump systems, which can cause high vibration, fatigue failures, frequent maintenance outages, and flow uniformity or product quality problems.

This paper describes a methodology to computer model pulsation using a combination of general-purpose and readily available, widely used software.

*Waterhammer Software* – "Classical Waterhammer" refers to the transient pressure spike and subsequent acoustic wave reflections that occur when a valve is suddenly closed in a long pipeline with established flow. Modern waterhammer software deals with all manner of transient pressure and flow dynamics in liquid piping networks. Any of several robust and benchmarked programs is suitable; see Ghidaoui, et al (2005). No matter the program of choice, henceforth it is simply referred to generically as "the waterhammer software". As will be seen in this paper, its use in pulsation analysis is atypical. For example when periodic once-per-rev flow forcing of a PD pump is input, the simulation is run until all transients die out, leaving only the steady-state pulsation.

*Modal Software* – Signal processing of high-speed digital time history data in the frequency domain can be done with many software platforms, including those for Experimental Modal Analysis (EMA), Data Acquisition, or general-purpose Statistical/Mathematical modeling. No matter the program of choice, it is henceforth simply referred to generically as "the modal software".

*Animation Software* – While animation is not necessary to do pulsation analysis, it is useful to visually show pressure and flow response throughout the piping network at acoustic natural frequencies and for PD pump flow forcing. The pulsation modal and forced response shapes in the piping network can be shown in 2D animation. This capability is typically included in EMA software.

*General Steps in Pulsation Analysis and Typical Results* – To computer model pulsation the following general steps are proposed.

STEP 1. Model the piping and process equipment system
STEP 2. Determine the natural frequencies and mode shapes of the system
STEP 3. Identify "worst-case" pump speeds that excite natural frequencies
STEP 4. Simulate operating pulsation response to PD pump flow forcing
STEP 5. Modify the system and reanalyze if pulsation is too high

To illustrate what pulsation results look like, Fig 2 shows the waterhammer model and Fig 3 the modal and forced response animation depiction of the Fig 1 physical system. Note that the Fig 3 animation model displays all piping, including branches, to be aligned along the X-axis (left-to-right). This is so that animation can be shown in the Y-axis direction to achieve a 2D representation of pulsation response throughout the piping network.

Figure 4 shows the forced response and animation of simulated pressure pulsation for the West pump running at 250 RPM. Maximum pulsation is about 6 PSI-PP or 10% of the mean suction pressure. Figure 5 shows pulsation for a lower speed, 215 RPM. Figure 6 shows pressure pulsation animation for these speeds using alternative animation software. Note that pulsation is much worse at the lower speed - 29 PSI-PP (47%)! Among the goals of the remainder of the paper are to answer questions such as, "Why is pulsation so much higher at a lower pump speed?", and "How does one determine this and gain a thorough understanding of the pulsation behavior in the system?"

*Why is Computer Modeling of Pulsation Only Now Entering the Public Domain?*

The mathematical components needed to computer model pulsation as described here are not new. Waterhammer and modal analysis have been extensively documented in the literature going back decades to the 1980's and earlier. But even today, pulsation analysis remains largely cloistered within a few high-tech engineering consulting firms. There are many reasons for this. One factor though is that these components tend to reside in very different places. Waterhammer analysis is a small specialty that resides in the Civil Engineering discipline. Modal analysis is a specialty too, but it resides to a large degree in Mechanical Engineering.

Detailed methodology for pulsation analysis is only now being documented in the manner presented here in large part because this is the first time the "dots are connected" linking these historically unrelated niche specialties.

*Target Audience for Performing Pulsation Analysis*

It is intended that all of the basic methodology and equations needed to facilitate computer modeling of pulsation be captured in this paper. In instances where spreadsheet calculations are suggested but not specifically shown in tables, the explicit equations are given to enable the needed spreadsheets to be created and customized. This, in combination with any of several waterhammer and modal software programs one may choose, should be sufficient to enable the example problem results shown here to be reproduced, and for the methodology to be applied to real-world process systems.

The successful practitioner though will also need solid academic foundations in specific areas beyond what is feasible to convey within this paper. These include dynamics governed by second-order linear differential equations (Thomson, 1972), waterhammer (Ghidaoui, et al, 2005), and digital signal processing (Vazquez, et al, 2012). These references provide excellent starting points to gain this knowledge. Many good short courses are also available, which are typically three to five days in duration.

## DETAILS OF PULSATION ANALYSIS

*Wavespeed*

Sonic velocity or "wavespeed" is a fundamentally important quantity in acoustics in any medium, whether through air with audible sound, through solids such as walls, insulation, etc., or through liquids such as in piping systems. In bulk liquids the wavespeed is determined by density, $\rho$, and Bulk Modulus, *B*. The latter is defined and approximated as follows, where *V* is liquid volume or specific volume, *P* is pressure and *s* is entropy.

$$B \equiv -V\frac{\partial P}{\partial V} \approx \left.\frac{-\Delta P}{\Delta V/V}\right|_{\Delta s=0} \qquad (1)$$

So, the less the volume changes for any change in pressure, the larger the value for *B* and the "stiffer" the liquid. The more the volume changes for any change in pressure the lower the value of *B* and the more "compressible" the liquid. Bulk Modulus, *B*, for liquids is analogous to Young's Modulus, *E*, for solids. Unlike Young's Modulus though, Bulk Modulus is non-constant and typically increases nonlinearly with pressure. For water at 70F and 1 atmosphere, $B \approx 310{,}000$ PSI. Tabulations of values for *B* for other liquids and conditions can be found in the literature.

The wavespeed, *a*, in the bulk liquid is given as follows.

$$a = \sqrt{\frac{B}{\rho}} \qquad (2)$$

Wavespeed in a pipeline will vary from this, though, depending on pipe wall flexibility as well as piping support conditions (c1).

$$a = \sqrt{\frac{B/\rho}{1+c1(BD/Et)}} \qquad (3)$$

The correction for wavespeed is accomplished in the denominator under the radical. Without this denominator term, Eq 3 is identical to Eq 2. Equations to evaluate c1 can be found in Wiley (1993), though in the examples to follow the author used the waterhammer software itself to evaluate the c1 values.

*Model the Piping and Process Equipment System (STEP 1)*

Modern waterhammer software utilize various approaches such as Method of Characteristics (MOC), Wave Plan or Wave Characteristic Method (WCM), Finite Difference, or others; see Ghidaoui, et al (2005) and others. The fundamentals of MOC, as well as some finer points important in pulsation analysis, are provided later. Simply though, MOC is a "lumped parameter" type modeling technique. This means that the actual continuous system is discretized or broken into segments. Each segment is of length, $L_i$, which is generally different for each pipe and vessel. The time-transient simulation is carried out in increments of time, *dt*, which is fixed for the simulation.

The relationship between these varying length segments, $L_i$, and the fixed time step, *dt*, is simple but important.

$$L_i = a_i * dt \qquad (4)$$

where $a_i$ is the wavespeed in pipe segment $L_i$.

The lowest frequency excited by the pump is called the 'base pulsation frequency', $f_b$. This is the pump speed, *N*, times the number of heads, *n*, assuming all cylinders are identical and equally spaced around a crank revolution.

$$f_b = nN/60 \qquad (5)$$

To do pulsation analysis, the time step and other parameters must be selected, for which three (3) criteria are proposed here. The first is Eq 4. The second criterion relates to the expected maximum frequency, $f_c$, of interest in pulsation. One rule of thumb is that $f_c$ be on the order of about six times the base pulsation frequency.

$$f_c \approx 6f_b = nN/10 \qquad (6)$$

Note that this rule of thumb is somewhat arbitrary. The analyst may have good reason to select a value different from this, either higher or lower. In the absence of knowledge of the range of frequencies that are problematic in the particular system being analyzed, the author has found Eq 6 to generally provide a conservatively high maximum frequency.

Once a value is assigned to $f_c$, no matter the means, the second criterion is that the time step $dt$ must be less than ½ of $1/f_c$. This time step is denoted $dt_a$.

$$dt_a = 1/(2f_c) \qquad (7)$$

For the particular five-headed plunger pump of Fig 1, operating at its maximum speed, Eqs 5, 6, and 7 can be evaluated.

$$f_b[Hz] = \frac{nN}{60} = \frac{5(280)}{60} = 23.3\ Hz$$

$$f_c[Hz] \approx 6f_b = 140\ Hz$$

$$dt_a = \frac{5}{nN} = \frac{5}{5*280} = 0.003574\ sec.$$

The third criterion is that the actual time step to be used in the waterhammer software, denoted $dt_b$, be chosen so as to reduce modeling errors to acceptable levels. The methodology to do this is explained as part of the following example.

*Develop the Basic Data to Input into the Waterhammer Software*

Fig 1 shows detailed data for the example problem, which can be input into the waterhammer software as-is. Typically, the waterhammer software contains logic to approximate the model parameters and suggest a step size to use, attempting to minimize errors. In the methodology proposed here though these decisions are made by the analyst and dictated to the waterhammer software, using Eqs 4, 6 and 7 and other criteria.

Table 1a shows a spreadsheet that accomplishes this for the Fig 1 system. The data and column identifications are as follows.

The constants are identified – step size per Eq 7, liquid density, viscosity, Bulk Modulus, and pipe material Young's Modulus
Col. 1: *i* = Element (pipe) number ( i = 1,2,…,9 for the 9 pipes in Fig 1)
Col. 2: *D* = Pipe inside diameter [in]
Col. 3: *t* = Pipe wall thickness [in]
Col. 4: *L* = Actual length of the pipe
Col. 5: *c1* – constant per Wylie [2] to account for effect of pipe support on $a_i$
Col. 6: $a_i$ = wavespeed calculated for pipe i; according to Eq 3
Col. 7: $L_i = a_i*dt$; according to Eq 4
Col. 8: *n* = integer closest to $L/L_i$
Col. 9: $L' = n*L_i$ = approximate line length used in the waterhammer program
Col. 10: error between pipe length actual (*L*) versus as-modeled (*L'*)

The values in Col's 1-4 are basic data transcribed from Fig 1. The value in Col 5 for c1, as mentioned earlier, is most easily acquired from the waterhammer software. The values in Col's 6 and 7 are simply Eq 3 and Eq 4. The values in Col's 8 and 9 are the approximate pipe length and number of segments to be used in the actual waterhammer software. The following are the only quantities that are input into the waterhammer software.

The system data values: *ρ, μ, B, and E*
For each pipe: *D, t, c1*, and *L'*

Now note the Col 10 "error" values. This is the error between the pipe length as-modeled versus actual. Pipes 2, 8, and 9 look to be terrible, but in reality these aren't the main concern. These are short elements and the effect of their length on overall dynamics is small. In the case of the dampeners (pipes 8 and 9), their shape can be adjusted while preserving the correct volume. The real concern has to do with the longer pipes, 1, 3, 4, 5, 6, and 7, where errors range from 4% to 8%. At this point alternative time steps, *dt*, are explored in order to reduce these errors. This is done in Table 2, in which the "long" pipes are identified and listed across the top row. The left-hand column of Table 2 list alternative time steps, *dt*, which are less than the maximum allowed according to Eq 7. For each of these pipes, the values for *n, L'*, and *Error* are calculated using the same equations as were used to generate Col's 8, 9, an10 of Table 1a. The right-hand column is the composite RMS error for all the long pipes. The Table 2-Inset plots this RMS Error as a function of Time Step, *dt*.

Based on Table 2, the time step choices that look attractive are 0.0017114 and 0.0006164 sec. These result in a ratio $dt_a/dt_b$ of 2.1 and 5.8, respectively. It will be seen later that this ratio should be greater than 3.0 to best represent the dynamics in the physical grid, thus favoring the 0.0006164 sec. value. For simplicity and smaller problem size though, the larger value $dt_b$ = 0.0017114 sec. is selected for the example problem in this paper. Table 1b is identical to Table 1a, but with this "improved" time step.

The smaller *dt* means the data file sizes will be larger by a factor of about 2.1 versus the maximum time step. Computational run times are likewise longer, but this is really not important. The main trade-off is model accuracy versus file sizes that must be managed in post-processing for natural frequencies. This all falls under the discretion of the analyst.

The data is now ready to enter into the waterhammer software. As mentioned, this includes the values *ρ, μ, B* and *E* for the system; and *D, t, c1* and *L'* for each of the nine pipes. The resulting model is used both for determining the acoustic natural frequencies and the steady-state forced responses.

*Wavelength, λ*

A later section will explore how to determine acoustic natural frequencies and mode shapes (STEP 2). First though an important dynamic property called the "wavelength" is introduced. Each natural frequency has associated with it a unique mode shape, which shows pulsation spatially throughout the piping network. When viewed in animation, also later in the paper, the mode shape for any particular natural frequency exhibits "nodes" and "anti-nodes". An anti-node is a physical location in the system where pulsation is relative maximum. Nodes, also called "zero-crossings", are locations where pulsation at that frequency is zero. For any acoustic natural frequency, one full sine-wave cycle in the mode shape is the wavelength for that frequency, given as follows.

$$\lambda_{ij} = a_i/f_{nj} \tag{8}$$

<u>*Model Error to "Maintain the Integrity of the MOC Grid"*</u>

Tables 1 and 2 are important tools to identify modeling parameters to minimize errors introduced just to make the physical system conform to certain mathematical requirements of the MOC. Additional error sources, such as wavespeed estimation, pressure loss characteristics, etc. are a separate matter. The errors managed in Tables 1 and 2 are only those that are imposed to "maintain the integrity of the MOC Grid". This will be explained further in the section on MOC. Very simply, if Eq 4 is met for every pipe segment in the network then the integrity of the MOC grid is maintained. It is as simple as that.

Since the fixed time step *dt* in Eq 4 applies to all pipes for the simulation, these errors for each pipe in the network must manifest themselves as "adjustments" to either the wavespeed or length. An important question is, "Does it make any difference in the simulation results where the errors reside?" The answer is 'No'. It does not make any difference. Table 1 puts the errors into pipe length. The waterhammer software used in this simulation puts the errors into wavespeed. (Actually, in this methodology, Table 1 supersedes the logic in the waterhammer software placing the errors into length). This can be shown with a simple example, which serves the dual purpose in the spirit of the tutorial to further illustrate the workings of Eqs 4-7 and exactly what Table 1 is doing. Consider within a piping network a particular pipe *i*, of length 42 ft and wavespeed 3,100 ft/s.

$a_i = 3{,}100\ ft/s$

$L = 42\ ft$

Suppose for example that for a natural frequency, $f_{nj}$, the wavelength $\lambda_{ij}$ corresponds exactly with this length. Applying Eq 8 this natural frequency can be determined as follows.

$\lambda_{ij} = L = 42\ ft$

$f_{nj} = {}^{a_i}/_{\lambda_{ij}} = \frac{3{,}100}{42} = 73.81Hz$

Now suppose the time step in the waterhammer model is 0.001953 s. Table 1 would yield the following model parameters and in turn, natural frequency and wavelength.

dt = 0.001953 s

$L_i = a_i * dt = 3{,}100 * 0.001953 = 6.0657\ ft$

*n* = 7

$L' = n * L_i = 42.383\ ft = \lambda_{ij}$ (length error = 0.912%)

$f_{nj} = \frac{a_i}{L'} = \frac{3{,}100}{42.383} = 73.143Hz$ (natural frequency error = 0.912%)

Putting the model error into length, according to Table 1, results in the natural frequency in error by 0.912%. Now suppose instead the correct pipe length is used and the wavespeed is adjusted. This results in the following in the waterhammer software.

*L' = L = 42 ft*

*n = 7*

$L_i$ *= 6 ft*

$a_i = \frac{L_i}{dt} = 3{,}072\ ft/s$ (wavespeed error = 0.912%)

$f_{nj} = \frac{a_i}{L'} = \frac{3{,}072}{42} = 73.143Hz$ (natural frequency error = 0.912%)

This simple example shows that it doesn't make any difference whether one adjusts wavespeed or pipe length in the pulsation analysis.

*Determine the Acoustic Natural Frequencies and Mode Shapes of the System (STEP 2)*

The nature of steady-state dynamics in any context is that the worst case conditions occur when excitation frequencies coincide with system natural frequencies. So, just running forced response cases at design speeds of the pump is not adequate. This is because there are many inaccuracies inherent in any computer model, both the nature and extent of which are unknown. If damping is low, which it is in pipeline acoustic systems, even small inaccuracies can mean that individual forced response simulations can greatly underestimate the severity of the dynamics that may occur.

There are various ways to determine the natural frequencies of a system and to quantify their relative severity or importance. In mechanical vibration a common technique in Experimental Modal Analysis (EMA) is to impact a structure with a hammer, instrumented to measure force. This is an easy and effective way to input wideband energy. Depending on where the impact is made the structure then vibrates at all or many of its natural frequencies across some range of interest. This is essentially the method proposed here. Since it is done analytically and not experimentally, though, it is called Computational Modal Analysis (CMA). But the mathematics are the same. Alternative flow forcing inputs can be synthesized, such as "swept-sine" and others, but this is beyond the scope of this paper.

In the waterhammer model of the Fig 1 system a sudden, brief flow spike is applied at the West Pump location. The time history of this appears in Fig 7a. A ZOOM of this would show *Q = 50 GPM* for exactly 2 time steps around time, *t=1 sec*, and *Q=36 GPM* for all other time steps before and after. A frequency analysis of this appears in Fig 7b.

*Digital Low-pass Filter*

Since frequencies above $f_c$ (140 Hz) are not of interest, this is the ideal point to filter out unneeded high-frequency content. This is done with something called a digital low-pass filter (LPF), which nominally filters out content above a cut-off frequency $f_c$ while passing content below it. The Fig 7a time-history impact is passed through a low-pass filter with 140 Hz cut-off frequency. The result is shown in Fig. 7c and FFT of which appears in Fig 7d. The Fig 7c forcing function would be a good choice for use in the waterhammer software for the simulation to determine the important natural frequencies. Though, in the example here Fig 7a is used. A pressure pulsation response at a location '$j$' is shown in Fig 8.

*The Frequency Response Function*

Next, a frequency-domain quantity is calculated in the modal software called the Frequency Response Function (*FRF*), the components of which are denoted $h_{ij}(f)$. This is a complex function, meaning it contains both magnitude and phase information. It relates the pressure response $P_j(f)$ at a point "$j$" in the system to a flow force $Q_i(f)$ at a point "$i$" that caused it. While alternative formulations are commonly used to actually calculate the FRF, the simple formulation is as follows.

$$h_{ij}(f) = {P_j(f)}\big/{Q_i(f)} \tag{9}$$

There are various ways to display the *FRF*. Magnitude and phase are displayed in Fig 9, which is called a 'Bode Plot'. The peaks in Fig 9a indicate acoustic natural frequencies. The 180$^{O}$ phase shift is also indicative of a natural frequency. The amplitudes at these frequencies indicate severity of expected pulsation response at location '$j$' to unit force input at location '$i$'. For example, at 36 Hz the FRF shows amplitude of 16 PSI/GPM and the force-to-response phase shift from +90$^{O}$ to -90$^{O}$. This means that a sinusoidal flow force of 1 GPM-0P (zero-peak) applied at the West Pump location will result in a pressure pulsation response at a location '$j$' in the system of 16 PSI-0P.

The FRF will be an important component of pulsation analysis in the future state of this technology. This is because the FRF identifies:

- Acoustic natural frequencies
- Response amplitudes to unit flow-force input throughout the system
- Which natural frequencies are most severe, meriting attention in design

Commercial waterhammer software even today though is just not set up to export sufficiently large data files of time history response output needed for FRF post-processing by the modal software. So until software engines are developed and incorporated into waterhammer software, some shortcuts are introduced in the current methodology. This is to simply FFT the responses at selected locations over several seconds after the impact. This is shown in Fig 10a, which reveals the same natural frequencies seen in Fig 9, but lacks good frequency resolution as well as amplitude and phase reference to the applied flow forcing.

The forgoing Computational Modal Analysis (CMA) is aimed at determining the acoustic natural frequencies and the modal pulsation response shapes (mode shapes) associated with them.

With the important natural frequencies now known, the next step in pulsation CMA is to perform forced response simulations for specific pump speeds.

*Identify "Worst-Case" Pump Speeds that Excite Natural Frequencies (STEP 3)*

Figure 10b shows a spreadsheet that identifies the various pump speeds for which integer multiples align with the natural frequencies, calculated as follows.

$$N_{ij}[RPM] = 60 * f_{ni}/j \tag{10}$$

where the left-hand column lists the natural frequencies, $f_{ni}$, from Fig 10a, and the run-speed multiples, $j$, appear across the top. Note that 35.88 Hz is by far the most significant one (the 5.1 Hz natural frequency is below $f_b$ even for the lowest possible run speed). This shows that 215.25 RPM will excite the 35.88 Hz natural frequency due to the 10-time run-speed component or twice $f_b$. So this will be the overall worst-case pump speed in terms of exciting pulsation.

The next step is to calculate the flow forcing behavior of the five-headed plunger pump described in Fig 1 and to do this specifically for 215.25 RPM.

*Calculate the Flow Forcing Time History for the PD Pump*

Volumetric Efficiency (VE) of a pump is defined as the ratio of the actual volumetric flow rate at suction conditions to a quantity called the "swept volume rate", $Q_{sw}$.

$$VE[\%] = {}^{Q_S}\!/_{Q_{sw}} \; x\; 100\% \tag{11}$$

$Q_S[GPM]$ = actual flow rate per cylinder at suction conditions
$Q_{Sw}[GPM] = V_{sw} * N$
$V_{sw}[GAL]$ = swept volume (stroke * plunger cross-sectional area)
$N\ [RPM]$ = pump speed

Note that the actual flow in the numerator is specified "at suction conditions". It is not unusual in PD pump applications for pressure on discharge to be much higher than on suction and for their densities, and hence volumetric flow rates, to be different by several percent. So, even if there is no leakage, volumetric efficiency will be less than 100% by virtue of liquid compressibility.

For this reason it is sometimes more convenient to express *VE* not using volume units but rather using mass units in the following manner.

$$VE(\%) = \frac{Mass\ Pumped\ per\ Stroke}{\rho_s * V_{sw}} \; x\; 100\% \tag{12}$$

It can be shown that for the ideal (no leakage) case, $VE_0$ can be written as follows.

$$VE_0(\%) = \frac{LA\rho_S(1+C_l) - LA\rho_d C_l}{LA\rho_S} \; x\; 100\% \tag{13}$$

where $C_l$ represents the clearance or "dead" volume as a percent of swept volume. This can be simplified as follows.

$$VE_0(\%) = 1 + C_l(1 - \rho_d/\rho_s) \; x\; 100\% \tag{14}$$

This can alternatively be expressed in terms of suction and discharge pressures and mean effective Bulk Modulus, $B_{eq}$.

$$VE_0(\%) = 1 - \frac{C_l(P_d - P_s)}{B_{eq}} \; x\; 100\% \tag{15}$$

It is also possible to approximate the discharge density in the following manner if a value for $B_{eq}$ during pressurization can be estimated.

$$\rho_d \cong \rho_s\left(1 + \frac{P_d - P_s}{B_{eq}}\right) \tag{16}$$

This is only approximate and care must be exercised in regard to estimating $B_{eq}$. As mentioned before, Bulk Modulus is non-constant and typically increases nonlinearly with pressure.

The time history of flow forcing, such as that shown in Fig 10a, is evaluated here using this ideal or leak-free model. To acquire this, one now needs only the slider crank equations and the pump mechanical and fluid data. The slider crank equations are those for plunger displacement and velocity.

$$x(t) = R(1 - cos\Theta) - L_{CR} + \sqrt{L_{CR}^2 - R^2 sin^2\Theta} \quad (17)$$

$$\dot{x}(t) = R\omega sin\Theta - R^2\omega \frac{sin\Theta cos\Theta}{\sqrt{L_{CR}^2 - R^2 sin^2\Theta}}\text{; where} \quad (18)$$

$\Theta = 2\pi ft = \omega t$ $\quad\quad R = L/2$

The valve opening positions for the no-leakage case can be shown to be as follows.

$$x_{svo} = L(1 + C_l \frac{\rho_s - \rho_d}{\rho_s}) \quad (19)$$

$$x_{dvo} = L(1 - \frac{\rho_s}{\rho_d})(1 + C_l) \quad (20)$$

For the current example problem then, these values are as follows.

| | |
|---|---|
| $P_s$ = 50 psig | $\rho_s = 0.87\ g/cc$ |
| $P_d$ = 7,000 psig | $\rho_d$ =0.9132 g/cc |
| $B_{eq} = 140{,}000\ PSI$ | |
| $C_l = 70\%$ | |

$VE_0 = 96.5\%$
$x_{svo} = 72.39\ mm$
$x_{dvo} = 6.03\ mm$

The flow-forcing time history for one head was determined in a simple spreadsheet (not shown) by applying these equations. Since this is a five-headed pump, the complete time history of flow forcing of Fig 11a is the sum of five of these time histories, each phase shifted $72^{O}$.

While leakage is outside the scope of this paper, it is well to consider it in modeling. Leakage generally results in the suction valve opening earlier, the discharge valve opening later, and worsening of pulsation.

*Simulate Operating Pulsation Response to PD Pump Flow Forcing (STEP 4)*

The animations of forced responses in vibration EMA are called “Operating Deflection Shapes” or ODS. In pulsation CMA the animations of pump forced pulsation response are called “Operating Pulsation Shapes” or OPS.

The OPS will look similar to the mode shapes for each natural frequency that the pump excites. But since the pump excites many harmonics of the base pulsation frequency, $f_b$, the OPS animations often appear more complex, containing multiple modal components. When a pump speed aligns closely with a single important natural frequency the OPS may be indistinguishable from the mode shape for that frequency.

A flow-forced response simulation was then done in the waterhammer software, applying Eqs 17-20 as discussed above. Recall from Fig 10b the speed 215.25 RPM was identified as the worst-case pump speed because this excites the 35.88 Hz natural frequency due to the 10-times run-speed or twice $f_b$ component. The time-history and FFT of this is shown in Fig 11. This flow forcing is applied at West Pump location J8 in Fig 2. High pressure pulsation was shown earlier in Fig 5, both graphically and in animation.

The other speed case done was shown in Fig 4 for operation at 250 RPM, which shows much lower pulsation. So the acoustic natural frequency analysis of Figs 9 and 10 shows how proximity of forcing frequencies and system acoustic natural frequencies result in much higher pulsation response for operation at 215 RPM than at 250 RPM.

Additional forced response simulations would be made to ensure that the worst-cases have been identified. Alternative designs can be analyzed in like manner.

The OPS animations discussed so far have only been OPS-Pressure as seen in Figs 4-6. Much can be learned by also looking at OPS-Flow as seen in Fig 12, again for the 215.25 RPM speed case. OPS-Pressure is shown in Fig 12a (identical to Fig 5b) and OPS-Flow in Fig 12b. Note that physical locations that are maxima (anti-nodes) for pressure pulsation are minima (nodes) for flow pulsation, and vice-versa. This is no accident. This is an important characteristic of plane-wave pulsation in piping networks. While worthwhile additional discussion in this area is beyond the scope of this paper, one element is mentioned here. A common and effective way to reduce pulsation is to locate orifices at strategic locations in the system. Such orifices are designed to introduce very small pressure drop, but enough to be effective in dampening pulsation. It was mentioned earlier that pipeline acoustics are systems that are generally very lightly damped. This means that excitation energy that aligns closely with acoustic natural frequencies result in very high pulsation, while even a small separation between excitation and natural frequencies results in much lower pulsation. The orifice is a singularly effective means to introduce damping. But the location selected for such an orifice is important. It is ideally located at an antinode for flow pulsation, which is a node for pressure pulsation. Such locations in the piping network are more easily identified by being able to see both pressure and flow pulsation animations such as shown in Fig 12.

The last step (STEP 5) is, if pulsation is too high for the "worst-case" pump speeds consider various design modifications and repeat STEPs 1-4 until an acceptable design is achieved.

## FUNDAMENTALS OF WATERHAMMER AND THE METHOD OF CHARACTERISTICS

Modern waterhammer simulation focuses on solving two equations for mass and momentum conservation. These equations can be found, for example, in Wylie (1993). Note that the equations here have been converted from head into pressure and volumetric flow rate into mass flow rate, and apply only to liquids.

Mass conservation:

$$\rho a^2 \frac{\partial V}{\partial x} + \frac{\partial t}{\partial t} = 0$$

Momentum conservation:

$$\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial V}{\partial t} + g * \sin(\alpha) + \frac{fV|V|}{2D} = 0$$

The two equations have two independent variables (*x* and *t*) and two dependent variables *(P* and *V*).

As discussed earlier the wavespeed, *a*, is the speed of acoustic wave propagation and fundamentally important in both waterhammer and pulsation. In piping it is influenced by both pipe wall flexibility and piping support. Methods to calculate or estimate wavespeed are common in the literature (Wylie, 1993).

The most common approach to solving the equations is to apply the Method of Characteristics (MOC). The solution methodology using MOC for waterhammer is well documented (Wylie, 1993) and will not be repeated here. In summary the preceding two partial differential equations are linearly combined and converted into four ordinary differential equations which can be integrated and solved analytically. The numerical solution is then based on finite difference approximation of the analytical solution.

### *Errors, Uncertainties and Limitations of the Method of Characteristics*

Wavespeed is commonly considered to have significant uncertainty associated with it. Some references (e.g., Wylie, 1993) assert the uncertainty to be as high as ±15%. The uncertainty in wavespeed is proportional to uncertainty in natural frequencies as seen in Eq 9.

Errors of 5-10% are not uncommon for at least some of the pipes for what is called time and distance step size. The time step size, *dt*, and pipe sectioning (length step size) are interdependent according to Eq 4, so must be chosen concurrently. These choices end up being fairly restrictive in order to maintain the integrity of the MOC and ultimately result in what is commonly known as the "MOC grid". This is facilitated within MOC software or in the current methodology by applying Eqs 4, 7, & 8, and implemented in Table 1. To achieve uniform time steps then some roundoff error in pipe sectioning (length step size) is inevitable. The roundoff can be attributed either to wavespeed or pipe length but it is more commonly attributed to wavespeed in MOC. What this means in plain language is that the estimated wavespeed for each pipe must be adjusted to an alternate value in order to satisfy the required sectioning from the MOC. The justification for this adjustment is commonly attributed to the uncertain wavespeed previously discussed.

General uncertainty captures the wide range of errors in input data common to all engineering calculations. These include centrifugal pump curves and possible degradation over time, transient flow profile from a multi-cylinder positive displacement pump, pipe wall scaling and wall roughness, valve loss values and position, equipment (e.g., heat exchangers) pressure loss values, flow rates and

pressures obtained from measurements, tank levels, fluid properties (temperature uncertainty will impact density and viscosity), etc. Similar to wavespeed uncertainty a complete evaluation of natural frequencies will check the sensitivity to these other uncertainties.
*Waterhammer Applications*

It is worth pointing out the different contexts in which the term **Steady State** is in common use in the disciplines of Civil Engineering (waterhammer) and Mechanical Engineering (fluid dynamics/pulsation).

- **Steady State** *pressure and flow* in the waterhammer context within Civil Engineering is the mean, constant pressure and flow state – flat-line over time – absent transients or dynamics of any kind. The MOC waterhammer software used in this study uses a solver based on the Newton-Raphson method to determine the steady state pressure and flow distribution.

- **Steady State** *pulsation* in the context of this paper is also absent any transients, but refers to the periodic or cyclic pressure and flow dynamics present in the system due to PD pump forcing. In Mechanical Engineering the solution to a system of second order linear differential equations is comprised of dynamics that die out over time (transient) and continue over time (steady state); the latter called *pulsation* here.

Historically, waterhammer simulation has been viewed as an area only to be evaluated by specialists in the field of waterhammer. This was due in part to the unavailability of robust commercial software solutions. As the commercial engineering software market has evolved and matured, robust and viable solutions for waterhammer simulation have become available. While these software solutions can never replace the expertise of waterhammer specialists, they have lowered the barrier to entry for non-specialists to be able to do productive work as a waterhammer analyst, especially when using supported commercial software.

Waterhammer software often uses a graphically-driven user interface with a “worksheet” and drag-and-drop modules as pictured in Fig 2, for example. Modules types primarily fall into two categories – Pipes and Junctions, some of which are discussed as follows.

*Pipe Modules*

Pipes have length, diameter, frictional characteristics (e.g., wall roughness) and acoustic characteristics (wavespeed, *a*, discussed previously). They also have elevation profiles (e.g., linear horizontal, linear vertical, linear diagonal or non-linear).

In addition, at the user’s option pipes can have valves and fittings. Valves and fittings can be “embedded” in the pipe so the pipe captures the hydraulic loss aspects of the valves and fittings. Note two highly important aspects of embedding valves and fittings in pipes during any waterhammer simulation:

- Only passive elements can be used – any element such as a valve which changes position during a transient is not a candidate for embedding and must be treated as a junction as discussed below.

- Equivalent length methods for hydraulic losses cannot be used – While acceptable for steady-state flow, artificially increasing the pipe length to account for increased pressure loss from valves and fittings will impact the acoustic propagation times. This is unacceptable under any and all circumstances. Such pressure loss increases must be handled by use of methods such as K factors which do not increase the pipe length.

Finally is the issue of non-flowing pipes. In steady flow modeling these are commonly neglected. In waterhammer modeling the effect of non-flowing pipes on transients and acoustic characteristics can be significant. Typically, the larger the volume of non-flowing pipes in relation to flowing ones, the larger the effect. In general, it is difficult to say for certain the impact of non-flowing pipes so they should always be considered at the beginning of the modeling process and then later neglected based on experience and judgment, rather than the opposite as is typical with steady-state flow simulation. The Figure 2 model includes one such non-flowing pipe as element P4 which connects to a dead end.

*Junctions*

Each pipe is initiated and terminated at a junction. In the Figure 2 model six different junction types are used:

- Reservoir (J10)
- Pump (J1, centrifugal in this model)

- Area Change (e.g., J2)
- Tee (e.g., J4)
- Dead End (J11)
- Assigned Flow (J8 and J9, which represent the PD pumps)

Junctions perform a variety of functions in a waterhammer simulation, but in general they capture the following aspects:

- Boundary conditions which can be steady over time or in fact change with time
  - known pressure (e.g., J10, steady pressure with time)
  - known flow (e.g., J8 and J9 PD pumps, flow varies in a pulsating transient manner)
- Active hydraulic elements
  - centrifugal pumps – which may run in a steady mode during the transient or themselves have transients due to loss of power, intentional shutdown, startup or variable speed controllers
  - valves – which may change position with time or may not (note valves which never change position during the transient of interest – for example, isolation valves around equipment – are best modeled as embedded elements as discussed in the Pipes section previous to this)
  - Check valves
  - Control valves
  - Surge suppression devices such as accumulators, surge tanks, relief valves, and vacuum breaker/air release valves
- Diameter changes
- Flow splits
- Elements that interact with a known pressure directly
  - Spray nozzle or sparger
  - Discharge valves with no pipe downstream (e.g., atmospheric discharge or submerged in liquid)
  - Weir

Note that some pipe system elements that one might at first treat as a junction element could better be represented as a pipe element. These would include elements that have significant length and associated acoustic interaction aspects. In fact this was implemented in the Figure 2 model where pipe P2 represents a flow meter and pipes P8 and P9 represent dampeners.

In summary, waterhammer software products which are based on the MOC have restrictions on time step that are imposed by the MOC. Further, time step size must be adequately small to resolve the desired frequency evaluation.

Two other time step issues exist. First, time steps may be driven to extremely small values based on the length of the pipes in the waterhammer model. Specifically, extremely short pipes in the model become the driver for time step size selection. These must be either avoided or modified to avoid significant impacts on model run time.

For example in steady flow modeling it is not uncommon for the analyst to use a fictitious, and physically very short, pipe element to connect different pipe system constructs such as a valve directly to a tee. The engineering analyst may select this pipe to be, for example, 0.1 inches in length, which has no perceptible impact on the calculation. This is not the case for waterhammer simulation. The short, fictitious pipe will have a dramatic and exponential impact on model run time. Thus short, fictitious pipes should be avoided. Further, suppose there is a very short physical pipe in the system. The impact on model run time has a similar impact as the short, fictitious pipe. The engineering analyst is typically advised to ignore such short pipes in the model or assume their physical length as represented in the model is significantly longer than the physical length. Robust waterhammer software assists both waterhammer novices and experts with making these choices. When doing so the time step can be increased to a value which is then proper for the frequency evaluation.

Second, the time step size must be short enough to capture the speed of important events during the anticipated transient. For example, a time step of 5 seconds is not adequate to understand the impact of a valve which closes in 1 second. Some time steps – certainly smaller than 1 second in this example – are needed. When in doubt, the engineering analyst should reduce the time step until the results are no longer visibly impacted. When focusing on frequency evaluation as in this current study this second time step consideration would rarely be applicable.

One of the interesting aspects of the MOC is that once all the previously discussed aspects of time step size are satisfied, then additional time step reductions offer virtually no value.

In more common finite-difference based computational approximations, reduced time step is considered to be advantageous because it allows more precise approximations to the derivative functions of continuous space. However, when the fundamental equations are solved by the MOC then that is not necessarily the case. If the resulting algebraic equations are linear (as is the case with waterhammer, see (Wylie, 1993) for example) then shorter time steps (which for MOC also mean shorter distance steps) offer no real value for increased accuracy. The engineering analyst then should focus on the maximum possible time step which satisfies all the other considerations previously discussed.

*Impact of Hydraulic Resistance on Waterhammer and Frequency Analysis*

In that a majority of industrial piping applications operate in the turbulent flow regime, it is common in many solutions of the governing equations that the hydraulic resistance is treated as constant during the transient simulation.

The basis for treating the resistance this way can be understood by considering a traditional Moody chart for pipe friction factor. For large Reynolds numbers one can see in Fig 13 a line not shown in all renditions of the Moody chart. This is the line which demarks on its right the zone of “Complete turbulence”. In some Moody charts this is referred to as the “Fully Rough” zone. To the left of this line is what Fig 13 calls the “Transition zone”. In the complete turbulence zone the pipe friction factor loses dependence on Reynolds numbers and for any given roughness value is essentially flat. On the other hand, the transition zone slopes upward as the Reynolds number decreases.

When the pipe flow is in the zone of complete turbulence and the friction factor loses dependence on Reynolds number, this is another way of saying that the friction factor is independent of flow rate. Further, when looking at the transition zone in Fig 13, it is clear that the friction factor dependence on Reynolds number (and hence flow rate) is relatively weak. Even over a Reynolds number change of two orders of magnitude the friction factor changes only by a modest amount. It is for this reason that treating pipe hydraulic resistance as a constant during a waterhammer transient has justification.

This raises a question. When the hydraulic resistance is treated as constant, which constant value should be used? The typical answer is that the hydraulic resistance obtained during the steady flow state should be assumed as constant during the transient simulation. A situation where the constant hydraulic resistance assumption clearly breaks down is when the flow is laminar or what Fig 13 calls the “critical zone” where the laminar/turbulent transition occurs. Here much large changes in friction factor with Reynolds are possible. Because a transient is occurring during waterhammer, it is clearly possible for the flow to begin in the zone of complete turbulence and as the flow rate decreases move into the transition zone and then into the laminar zone. If the time duration in the laminar zone is relatively short then the impact of the constant hydraulic resistance assumption is minimized. However, if the flow becomes laminar for a longer duration then the assumption can break down.

A second situation can occur which complicates the reasoning process outlined in the preceding discussion. What if the waterhammer simulation of interest begins with stagnant flow and the application is to understand the transient when the flow is started? A typical application for this would be a pump startup transient. The issue here is that with zero flow in the steady-state, a Reynolds number does not exist and a unique hydraulic resistance (i.e., friction factor) does not exist. What constant hydraulic resistance is to be assumed for the transient simulation?

One solution to the two issues is for the hydraulic resistance to be treated as a variable and updated based on changes in Reynolds number (flow rate). While this was problematic in the early days of digital computing because of computer memory limitations, today it is straightforward to achieve in most commercial software such as that used in this study.

If this is the case, then why don’t all commercial software applications just always use the variable resistance approach? The answer is that constantly updating the resistance every time step does take extra computing time. Experience suggests it makes a simulation take about three times as long. And this is to address a situation that occurs in perhaps only 1% of situations.

A second solution which is especially applicable to the situation when the flow begins as stagnant is to assume that the constant hydraulic resistance should be based not on the zero steady-state flow (which does not have a hydraulic resistance) but on some typical turbulent flow value. Choosing a basis velocity for the Reynolds number of 5 ft/sec and obtaining a friction factor for that Reynolds number will frequently be adequate for this situation.

Bringing this back to the case of pulsating flow, the issue of constant hydraulic resistance will not be significant unless there is a large range of flow rates or the regime is laminar or transitional for part of the time. In this study it was not accounted for and likely would not be significant. A future study would be recommended to confirm this.

Experience shows that overall the impact of hydraulic resistance changes on the waterhammer transient results is of second order. That is why it is common for hydraulic resistance to be “relocated” in a model. This is what happens, for instance, when embedded fittings and losses are used as discussed earlier. These embedded values in reality get changed from individual point values of resistance into increased pipe friction and are thus spread out along the entire pipe. The benefit obtained with this approach is the ability to take longer time steps. Because it is commonly the experience that the ultimate results are not changed significantly by embedding valves and fittings, the value of using larger time steps (which results in much shorter simulation run times) takes on the higher priority. The impact of hydraulic resistance modification (relocation and/or embedding in pipes) on pulsation studies such as this is not well understood and a future study is recommended to clarify this further.

## FUNDAMENTALS OF DIGITAL SIGNAL PROCESSING AND APPLICATIONS IN PULSATION

Experimental Modal Analysis (EMA) is often aimed at mechanical structures to determine natural frequencies and their vibration response behavior. These "Mode Shapes" show vibration spatially across the structure at each natural frequency. "Anti-nodes" are physical locations where vibration is relative maxima, while "nodes" are locations where vibration is zero. Digital Signal Processing or DSP comprises the mathematics and statistics used, for example, to analyze digitized data in the performance of EMA. Vazquez, et. al. (2012) and others provide good basic introductions to these fields, which are otherwise beyond the scope of this paper.

The mathematics of DSP are important in pulsation as well, so a discussion of basic terminology and relationships is included here.

Waterhammer analysis is performed in the time domain and is carried out with a fixed step size or **Time Resolution**, dt [sec.]. The inverse of this is called the **Sample Rate**, SR [Hz]. Digital data is processed in "blocks". The time span of each block is called the **Time Window**, T [sec.] and the number of digital data samples contained within this time is called the **Block Size**, BS.

Being able to view the signal in the Frequency Domain is as important as the Time Domain. So the time history data must be converted to frequency and phase representation. This too is digital, which means that it has a finite **Frequency Range**, F [Hz], **Frequency Resolution**, df [Hz], and number of frequencies called **Lines**, L.

Notice there are seven (7) terms above that are bolded. This provides the context for understanding what is meant by the various terms commonly used in dynamics and in signal processing. These seven parameters are related in the following manner.

Time Domain:

$$dt = 1/SR \tag{21}$$
$$T = BS * dt = BS/SR \tag{22}$$

Frequency Domain:

$$L \leq BS/2 \tag{23}$$
$$F \leq SR/2 \tag{24}$$
$$df = 1/T = SR/BS \tag{25}$$

Many frequency analysis algorithms are available, such as, but not limited to, the Discrete Fourier Transform (DFT) and Fast Fourier Transform (FFT). The term FFT is used here because of its common usage, but applies generically to any means chosen to perform frequency analysis. The inequalities in the above equations are treated as equalities in DSP.

Another qualification that some algorithms may impose is that Block Size be an integer power of 2.

$$BS = 2^m, \text{ where } m \text{ is any integer} \tag{26}$$

For example, for $m$=10, $BS$ = 1,024 (or 1K); for $m$=11, $BS$ = 2,048 (or 2K); for $m$=15, $BS$ = 32,768 (or 32K); etc. It is conventional to express block size with the "K" abbreviation.

The above equations also show that of the seven (7) parameters, only two (2) are independent. So, one must choose exactly two, but not just any two. One must choose no more and no less than one value from two of the following three groups.

1. *F*, *SR*, or *dt*
2. *df* or *T*
3. *BS* or *L*

In the example problem explored here the waterhammer model has 47 pipe segments. This is the sum of the values in Col 8 in Table 1b. A pipe with $n$ segments has $n+1$ pressure response points. So the 9 pipes in this example have 56 points in the MOC model (47 +

9). The time step chosen was $dt_b$ = 0.0017114 sec. This means that *SR* = 584.3 Hz and *F* = 292 Hz. Exactly one more parameter must still be chosen. In the methodology proposed here the second parameter can be either Block Size *BS*, or Time Window *T*. If the Eq 26 limitation applies to the user-chosen modal software, which it may or may not be, then BS is the second DSP parameter. It is chosen in two different ways.

- *Rough Estimates of Natural Frequencies and Pipe Locations that Participate in the Mode Shapes* – Small *BS* say 1K – 2K (1,024 – 2,048) is selected and DSP is done on many if not all physical locations. This was done to generate Fig 9a.
- *Accurate Identification of Natural Frequencies* – Large *BS* is needed to achieve fine frequency resolution to distinguish closely spaced modes and also to accurately identify the pump speeds that excite them. This requires only a few physical locations to be included in post-processing, but large blocks, say, 32K, 64K, or more. This was done to generate Fig 8.

In the methodology proposed here, DSP is done twice, in successive iterations in this manner, on the same data. This is necessary due to the file size and data transfer limitations of existing waterhammer software. As mentioned earlier, more efficient software engines need to be developed and incorporated into the waterhammer software, at which time all physical locations with large block size (long time window) will be done at once.

The first iteration of DSP is to extract results from the waterhammer software for all, or a representative number of points using a small *BS*. The FFTs of these data records are then plotted, which provide coarse estimates of the important natural frequencies. Animations of the system response at each of these frequencies in a manner like Fig 10 then provides at least a coarse visual understanding of the mode shapes – "coarse" because any closely-spaced modes that may exist become melded together. The following explores how this might work for the current example.

*p* - # of physical points included in post-processing = 56 points
*dt* = 0.0017114 sec
*BS* = 2K (2,048)

*SR = 1/dt* = 584.3 Hz
*F = SR/2* = 292 Hz
T = BS*dt = 3.505 sec.
*Δ-RPM* = resolution in pump speed = 60/*T* = 17.12 RPM
*FS* = file size (real values) = *p*T/dt* = 114,690

The second iteration of DSP is to identify one or more physical locations in the system that at least 'participate' in the important natural frequencies identified in the first iteration. A large block size (and time window) is extracted from the same waterhammer simulation. This small number of records (as few as 1) are FFT'ed to identify the important natural frequencies with fine resolution. Parameter settings and file sizes for the current example might be as follows.

*p* = 1
*dt*, *SR*, and *F* are the same as above for the first iteration of DSP
*BS* = 64K (65,536)

*T = BS*dt* = 112.16 sec.
*Δ-RPM* = resolution in pump speed = 60/*T* = 0.53 RPM
*FS = p*T/dt* = 65,536

By comparison if all 56 physical points are post-processed for the full 112.16 second time window, the file size would be about 3.7 million real values, a task 20-times that of this two-iteration process, which achieves the same results.

*Signal Aliasing*

In general, post-processing should be done on nothing less than the full time-transient output to avoid a problem called "signal aliasing", which corrupts the data making it unusable. Signal aliasing affects both time-history and frequency-domain data when the sample rate is inadequate to characterize high frequencies present in the actual signal.

To illustrate, consider the solid BLACK sinusoidal waveform of Fig 14. It has zero-peak amplitude of 10 and a frequency of 125 Hz. This could be output from the waterhammer software done at a high data rate. If simulation results are output only every 10 data points, one might receive data at only the RED dots shown. If processing is done on this decimated data, one sees only a 12 Hz sinusoidal behavior. "Aliasing" of the signal has occurred. At this point the data is corrupt and unusable. There is no way to fix the data after aliasing has occurred. The data must be discarded the original high-speed data reacquired.

In DSP an important term, called the Nyquist Criterion, states that in order to characterize both amplitude and phase of a periodic signal it is necessary to sample at a rate greater than twice the highest frequency present.

For any given time-domain data, whether analog or digital, the highest frequency of interest, $f_c$, is first identified. The signal is then passed through a low-pass filter, again analog or digital, set to this frequency. The signal can then be digitized with a sample rate of greater than 2*$f_c$.

Some waterhammer programs offer the option to provide output less than every time step (once every two or more time steps). This is called "decimating" the data. For example, decimation 5:1 means passing one value, discarding the next 4, passing one value, discarding the next 4, and so on. If the flow force input used was not low-pass filtered, such as the time histories of Fig 7a for impact or Fig 15a for PD pump, then the full $1/dt_b$ data rate must be post-processed, with no decimation. If the flow forcing is low-pass filtered such as shown in Fig 7c for impact and Fig 15c for PD pump flow forcing, then it is safe to decimate the output from the waterhammer software as follows.

$$\delta_t = INT\left(\frac{dt_a}{dt_b}\right) \tag{27}$$

$$\delta_t = INT\left(\frac{0.003574}{0.0017114}\right) = INT(2.09) = 2$$

So in this instance, again assuming the forcing function was low-pass filtered with cut-off frequency $f_c$, the time history output from the waterhammer software may be safely decimated 2:1. This means that every other sample is kept and every other discarded. Realize also that if the time history data is decimated 2:1, then in post-processing the sample rate is halved and the time step doubled.

*Spatial Aliasing*

If the physical grid of points representing the piping network is too coarse to depict animation of high-frequency, short-wavelength mode shapes or OPS, then something called "spatial aliasing" has occurred. Signal aliasing, as discussed above, occurs when the time-domain data rate is less than twice the highest frequency present in the signal. Spatial aliasing occurs when there are too few points in the physical animation grid.

This can be challenging in vibration EMA, where the physical grid of measurement points is often defined simply intuitively. The testing yields vibration data with good high frequency content, but the physical grid may not be detailed enough to display that complexity in animation. So high frequency modes may appear in animation incorrectly as lower frequency ones – or more often simply appear chaotic.

Pulsation CMA carried out as described here enjoys a distinct advantage in this regard. This is because the physical grid of points describing the piping network is determined hand-in-hand with the relevant DSP parameters of *F*, *SR*, and *dt*. So if the animation model contains all of the physical points in the MOC model, then spatial aliasing does not occur, meaning that the MOC pipe sectioning will produce two physical points per waveform for the highest frequency 1/(2**dt*). It turns out though that while two points per waveform is adequate in the FFT algorithm to characterize amplitude and phase of a sine wave, the human eye requires more points. A rule of thumb is that it takes about 6 points or so per waveform in the physical grid to render smooth sine wave animation as viewed by the human eye. So for animation purposes, the value $dt_b$ chosen for waterhammer modeling should not only provide a small composite error from Table 2 but also be on the order of 1/3 or less of the Eq 7 value of $dt_a$. Therefore, if the flow forcing is low-pass filtered such as shown in Fig 7c for impact and Fig 15c for PD pump flow forcing, then it is safe to decimate the physical grid spacing as follows.

$$\delta_s = INT\left(\frac{dt_a}{3*dt_b}\right) \tag{28}$$

$$\delta_s = INT\left(\frac{0.003574}{3*0.0017114}\right) = INT(0.7) = 0$$

What this says is that for the time step $dt_b$ used in the waterhammer software, not only can the physical grid not be decimated, it is not even adequate to provide at least 6 points per waveform for the highest frequency $f_c$. In order to have achieved this it would be necessary to use a smaller step size, such as point "C" in Table 2.

## CONCLUSIONS

The 5-STEP methodology for pulsation analysis proposed at the outset can be performed as follows.

*STEP 1.* *Model the piping and process equipment system*

a) Draw a picture of the piping network and record basic data like Fig 1

b) Identify the possible speed range and expand it by at least 15% on each end to account for all manner of "modeling errors"; more if uncertainty is great around fluid compressibility or other basic data

c) Based on this maximum "effective" pump speed, determine $f_b$ (Eq 5), $f_c$ (Eq 6 or other criteria) and $dt_a$ (Eq 7).

d) Make a spreadsheet like Table 1a and enter the system data values *dt, ρ, μ, B*, and *E*

e) Number each pipe in the system (Col 1) and enter the values for *D, t*, and *L* in Cols 2, 3, and 4.

f) Begin to build the model in the waterhammer software like Fig 2 and enter the system and pipe basic data. Let the waterhammer software calculate c1; enter these values in Table 1a (Col 5)

g) Work back and forth between Table 1a and the waterhammer software for the remainder of STEP 1

h) Use the following equations to fill out the remaining columns in Table 1a automatically

$$a = \sqrt{\frac{B/\rho}{1+c1(BD/Et)}} \qquad \text{(Col 6)} \qquad (3)$$

$$L_i = a_i * dt \qquad \text{(Col 7)} \qquad (4)$$

$$n = MAX(1, INT\left(\frac{L}{L_i}\right)) \qquad \text{(Col 8)}$$

$$L' = n * L_i \qquad \text{(Col 9)}$$

$$\epsilon_i = \left|\frac{L'}{L} - 1\right| \qquad \text{(Col 10)}$$

i) For volumes like the pulsation dampener, first evaluate c1 and wavespeed in the same manner as for pipes. Then fix wavespeed and calculate an effective diameter to give the correct volume for length *L'*

j) To reduce the Col 10 errors, identify the important (long) pipes and create Table 2 for step size *dt* ranging downward from the value per Eq 7.

$$RMS[\%] = \sqrt{\frac{\Sigma(L' * \epsilon_i^2)}{\Sigma L'}}$$

k) Identify and denote as $dt_b$, a value from Table 2 that is 1/3rd or smaller of the Eq 7 value in Table 1a, and also achieves a low RMS error.

l) Create a Table 1b identical to Table 1a, but using the "improved" value for step size, $dt_b$. The "effective" diameter of each volume element must also be adjusted, manually or automatically to provide the correct volume

m) Use the Table 1b, Col 9 values *L'* in the waterhammer software

n) Allow the waterhammer software to discretize the model and suggest *dt*. Unless there is a typo or other error it will give you your value for $dt_b$.

STEP 2. *Perform natural frequency simulations in the waterhammer software and post-process in the modal software*

a) Create an impulsive force at the pump location with fixed mean flow for all time except for a larger value early in the simulation of duration 2*$dt_b$. Plot this and an FFT of it like Fig 7a and 7b. Low-pass filter (LPF) this impulse time history and FFT it. Plot these like Figs 7c and 7d

b) Perform a waterhammer simulation with this low-pass filtered time history at the pump location. Carry out the simulation until the transient response has died out such as seen in Fig 8

c) Copy into the modal software the pressure response after impact for a "short" time block (try BS in the range 1,024 to 4,096 and adjust as needed). Do this for all locations or an adequate number and spacing in the animation grid to avoid spatial aliasing – then plot superimposed the FFTs of these like Fig 10a

d) Animate the modal behavior at each important acoustic natural frequency identified like Fig 10c; and choose one or two locations in the system that "participate" in all of them

e) Copy into the modal software the pressure response from just before impact at least until the transient has died out. Select *BS* based on desired frequency resolution. For example, 1 RPM pump speed resolution requires *T*=60 sec., *df* = 1/60 Hz, *BS* = *T/dt.*

f) FFT these data records and plot like Fig 10a to identify natural frequencies accurately.

g) Alternatively or additionally to Step 2f, FFT the impact also with the same settings as the response and generate the FRF

$$h_{ij}(f) = {P_j(f)}/{Q_i(f)} \qquad (9)$$

h) Animate the modal behavior at each important acoustic natural frequency to learn and understand the how pressure pulsation is distributed throughout the network for each natural frequency.

i) The most conservative approach is to process all time history results from the waterhammer software at the full $1/dt_b$ sample rate to avoid "signal aliasing" errors. The experienced practitioner, who has correctly applied the LPF to the impact and flow forcing input in accordance with the methods described above, can decimate the time history data and the physical grid spacing according to the following, respectively.

$$\delta_t = INT\left(\frac{dt_a}{dt_b}\right) \qquad (27)$$

$$\delta_s = INT\left(\frac{dt_a}{3*dt_b}\right) \qquad (28)$$

STEP 3. *Identify "worst-case" pump speeds that excite natural frequencies* – Equation 10 gives the speeds for which integer multiples of $f_b$ align with acoustic natural frequencies; see also the table in Fig 10b. A speed will appear in any cell of this table when Eq 9 produces a value that falls within the range identified in STEP 1a

$$N_{ij}[RPM] = 60 * f_{ni}/j \qquad (10)$$

*STEP 4. Simulate PD Pump Flow Forcing at the "worst-case" pump speeds*

a) Create PD pump flow forcing time histories for these run speeds in a spreadsheet (not shown) using Eqs 14, 17-20

b) Plot this and its FFT like Fig 15a and 15b. Pass the Fig 10a time-history through a LPF exactly as was done for the Step 2 impact event. Plot this and its FFT like Fig 10c and 10d.

c) Enter the low-pass filtered periodic flow-forcing into the waterhammer software for a length of time of $1/f_b$, and indicate as "repeating".

d) Carry out the simulation for enough time for the transients to die out, leaving only the steady-state pulsation

e) Copy into the modal software pressure and flow results for all points in the system for a length of time equal to $1/f_b$, beginning after the transients have died out – plot these results and view animations like Figs 4-6 and 12.

f) Repeat 4a), 4b), and 4c) for other worst-case pump speed to determine overall worst-case predicted pulsation levels

STEP 5. *If pulsation is too high, consider various system modifications to reduce levels and reanalyze* – Possible measures include but are not limited to addition/modification of dampeners, orifices, lines lengths and diameters, pump displacement and speed, and others.

**ACRONYMS**

| | |
|---|---|
| CMA | Computational Modal Analysis |
| DFT | Discrete Fourier Transform algorithm to calculate frequency content |
| DSP | Digital Signal Processing |
| EMA | Experimental Modal Analysis |
| FFT | Fast Fourier Transform algorithm to calculate frequency content |
| FRF | Frequency Response Function; with individual components denoted $h_{ij}$ |
| MOC | Method of Characteristics |
| ODS | Operating Deflection Shape |
| OP | Zero-Peak amplitude (½ of PP) |
| OPS | Operating Pulsation Shape |
| PD | Positive Displacement (typ. plunger pump) |
| PP | Peak-to-Peak amplitude (2 * 0P) |
| VE | Volumetric Efficiency |
| WCM | Wave Characteristic Method |
| WH | Waterhammer |

**UNITS ABBREVIATIONS**

| | |
|---|---|
| CP | centipoise |
| GPM | gallons per minute |
| g/cc | grams per cubic centimeter |
| Hz | Hertz or samples per second |
| in | inches |
| PSI | pounds per square inch |
| RPM | revolutions per minute |
| rad | radians |
| s | seconds |
| sec | seconds |

**NOMENCLATURE**

| | | |
|---|---|---|
| A | in² | cross-sectional area of plunger of PD pump |
| a | ft/s | wavespeed or sonic velocity in fluid |
| B | PSI | bulk modulus of fluid |
| BS | - | block size in DSP = number of samples within the block |
| $C_l$ | % | clearance ratio of cylinder; dead volume/swept volume |
| c1 | - | wavespeed correction factor per Wylie [2] |
| D | in | inside diameter of pipe |
| $df$ | Hz | frequency resolution in DSP |
| $dt$ | sec | time resolution in DSP or step size in waterhammer analysis |
| $dt_a$ | sec | time step per Eq 7 |
| $dt_b$ | sec | time step used in the waterhammer software |
| $F$ | Hz | frequency range in DSP |
| $f$ | - | friction factor - Darcy |
| $f$ | Hz | frequency |
| $f_b$ | Hz | base pulsation frequency generated by the PD pump |
| $f_c$ | Hz | maximum frequency of interest |
| $g$ | ft/s² | gravitational constant |
| K | - or Hz | multiple of 1,024; eg.: BS = 32K = 32,768 |
| $L$ | - | lines of frequency resolution in DSP |
| $L$ | mm | pump stroke length |
| $L$ | ft | actual physical length of pipe |
| $L'$ | ft | "adjusted" pipe length to be used in MOC model simulations |
| $L_{CR}$ | mm | connecting rod length of pump slider crank |

| | | |
|---|---|---|
| $L_i$ | ft | length of pipe segment modeled in MOC; $L_i = L'/n$ |
| $h_{ij}$ | PSI/GPM | complex FRF component at "j" due to unit flow forcing at "i" |
| $N$ | RPM | run speed of PD pump |
| $m$ | - | any integer |
| $n$ | - | number of elements a given pipe is divided into in MOC model |
| $n$ | - | number of cylinders of the plunger pump |
| $P$ | PSI | pressure |
| $p$ | - | number of physical points in the animation model |
| $Q$ | GPM | flow rate |
| $R$ | mm | radius of slider crank, $R = L/2$ |
| $SR$ | Hz | sample rate in DSP |
| s | | entropy |
| $T$ | sec | time window in DSP |
| $t$ | sec | time |
| $t$ | in | wall thickness of pipe |
| $V$ | $in^3$ | volume |
| $V_{sw}$ | gal | cylinder swept volume |
| $x$ | ft | distance along a pipe |
| $x$ | mm | plunger position relative to BDC |
| $\dot{x}$ | mm/s | plunger velocity |
| $x_{dvo}$ | in | plunger position when discharge valve opens |
| $x_{svo}$ | in | plunger position when suction valve opens |
| | | |
| $\alpha$ | rad | slope of pipe |
| $\delta_s$ | | spatial decimation factor |
| $\delta_t$ | | temporal decimation factor |
| $\Delta Q$ | | heat transfer |
| $\lambda$ | ft | wavelength |
| $\Theta$ | rad | crankshaft rotational position relative to BDC |
| $\rho_d$ | g/cc | fluid density on discharge |
| $\rho_s$ | g/cc | fluid density on suction |
| $\mu$ | CP | viscosity |
| $\omega$ | rad/s | frequency ($\omega = 2\pi f$) |

## REFERENCES


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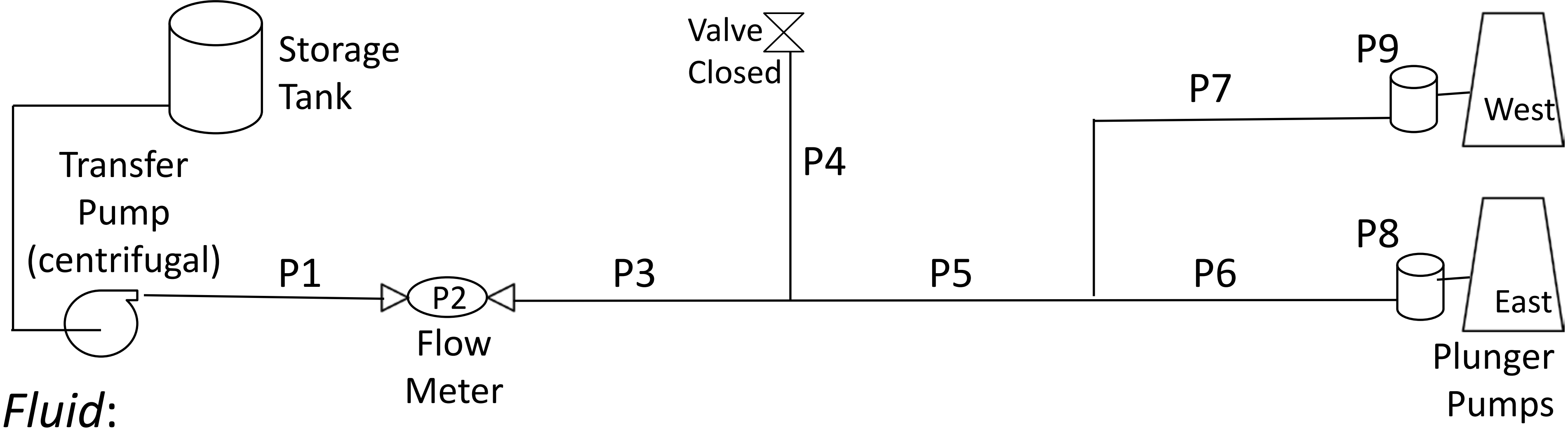



*Fluid*:

- ρ = 0.87 g/cc
- μ = 0.95 CP
- B = 140,000 PSI
- Ps = 50 PSIG
- Pd = 7,000 PSIG

*Plunger Pumps (2)*:

- Quintaplex (5 heads, 72$^o$ CA)
- 40mm ϕ x 75mm stroke
- N = 120 to 280 RPM (VFD)
- Cl = 70%

*Piping & Process Equipment*:

- P1 – 48 ft. x 3.26"D; 0.120" wall
- P2 – 3.2 ft. x 1.682"D; 0.109" wall
- P3 – 44 ft. x 3.26"D; 0.120" wall
- P4 – 22.5 ft. x 1.682"D; 0.109" wall
- P5 – 50 ft. x 3.26"D; 0.12" wall
- P6 – 34.5 ft. x 2.157"D; 0.109" wall
- P7 – 46.5 ft. x 2.157"D; 0.109" wall
- P8 & P9 – 30 gal. pulsation dampeners

Fig 1. SAMPLE PD Pump Suction System for Pulsation Analysis

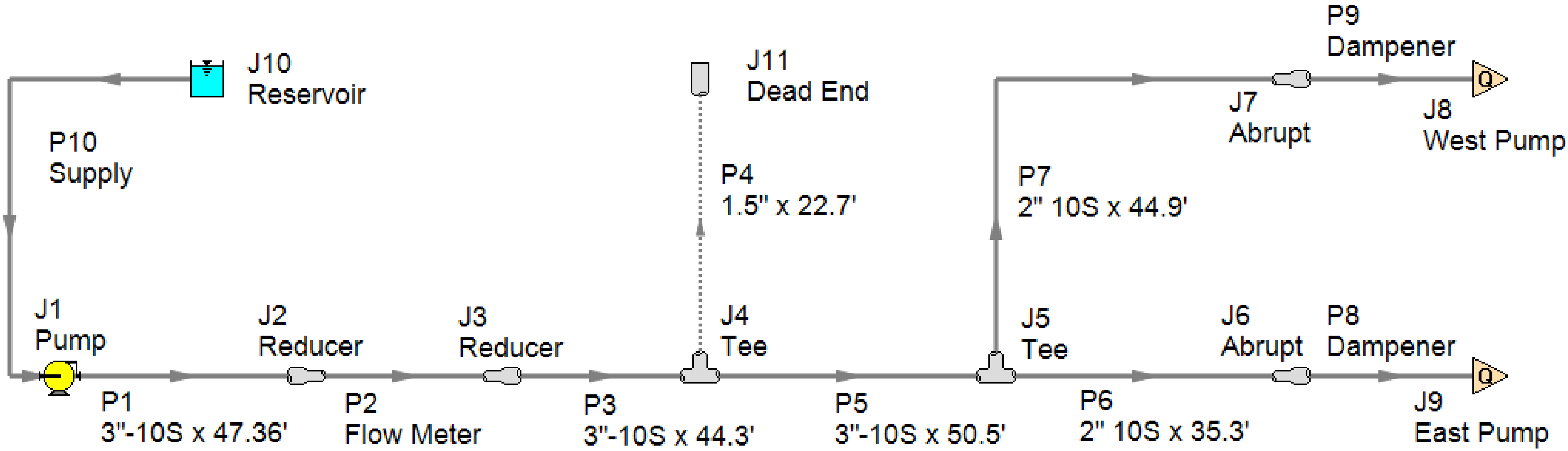



Fig 2. WATERHAMMER Model of Fig 1 Sample Problem – NOT to Scale

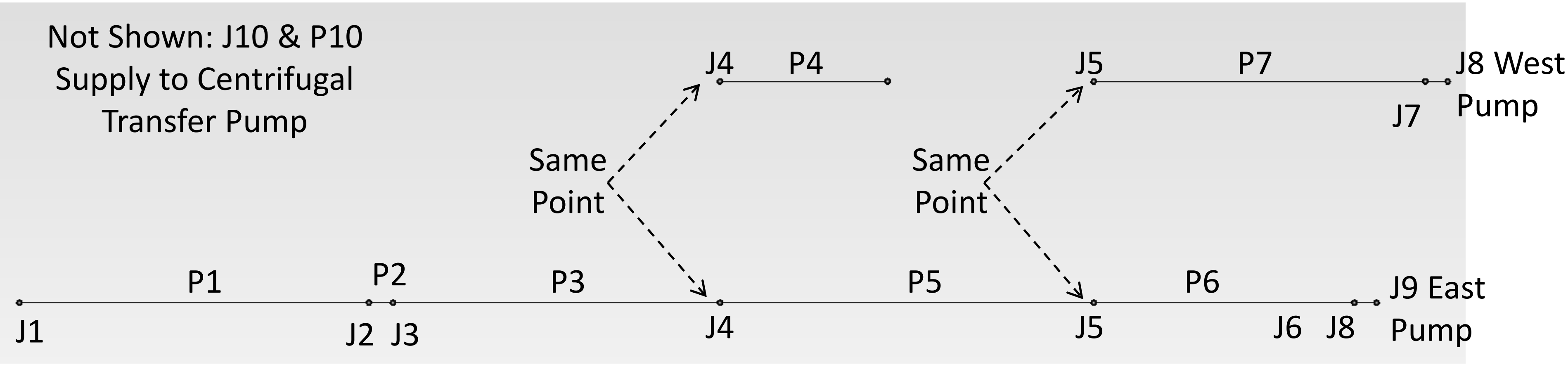



Fig 3. AMINATION Model of Fig 1 Sample Problem – To Scale in "X" (length) Dimension

*Basic Data for Fig 1 Piping Network and Simulation Step Size, dt*

| | | | |
|---|---|---|---|
| *dt* | 0.0035714 | [sec.] | desired time step in the waterhammer software |
| *ρ* | 0.87 | [g/cc] | liquid density |
| μ | 0.95 | [CP] | viscosity |
| *B* | 140,000 | [psi] | liquid bulk modulus |
| *E* | 2.83E+07 | [psi] | pipe material elastic modulus |

* - Enter wavespeed, *a* , directly into waterhammer software for volumes such as Dampener

| *Basic Data for each Pipe from Fig 1* | | | | *WH Software* | *Eq 3* | *Eq 4* | *Integer L/Li* | *n*Li* | *Error L' vs L* |
|---|---|---|---|---|---|---|---|---|---|
| col. 1 | col. 2 | col. 3 | col. 4 | col. 5 | col. 6 | col. 7 | col. 8 | col. 9 | col. 10 |
| Pipe i | D [in] | t [in] | L [ft] | c1 | a [ft/s] | Li [ft] | n [-] | L' [ft] | error |
| 1 | 3.26 | 0.12 | 48 | 1.06 | 3,233.2 | 11.547 | 4 | 46.188 | 3.8% |
| 2 | 1.682 | 0.109 | 3.2 | 1.06 | 3,323.9 | 11.871 | 1 | 11.871 | 271.0% |
| 3 | 3.26 | 0.12 | 44 | 1.02 | 3,240.8 | 11.574 | 4 | 46.297 | 5.2% |
| 4 | 1.682 | 0.109 | 22.5 | 1.11 | 3,318.1 | 11.850 | 2 | 23.700 | 5.3% |
| 5 | 3.26 | 0.12 | 50 | 1.06 | 3,233.2 | 11.547 | 4 | 46.188 | 7.6% |
| 6 | 2.157 | 0.109 | 34.5 | 1.084 | 3,285.9 | 11.735 | 3 | 35.205 | 2.0% |
| 7 | 2.157 | 0.109 | 46.5 | 1.084 | 3,285.9 | 11.735 | 4 | 46.940 | 0.9% |
| 8 | 8.439 | 0.188 | 3 | * | 2,890.9 | 10.325 | 1 | 10.325 | 244.2% |
| 9 | 8.439 | 0.188 | 3 | * | 2,890.9 | 10.325 | 1 | 10.325 | 244.2% |

TABLE 1a. Details of Waterhammer Modeling Sample Problem - *dt* = 0.0035714 sec. (per Eq 7)

| Pipe # | 1 | | | 3 | | | 4 | | | 5 | | | 6 | | | 7 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| L [ft] | | 48 | | | 44 | | | 22.5 | | | 50 | | | 34.5 | | | 46.5 | | |
| a [ft/s] | | 3,233.2 | | | 3,240.8 | | | 3,318.1 | | | 3,233.2 | | | 3,285.9 | | | 3,285.9 | | RMS |
| dt | n | L' | err | n | L' | err | n | L' | err | n | L' | err | n | L' | err | n | L' | err | ERR |
| 0.0035714 | 4 | 46.188 | 3.8% | 4 | 46.297 | 5.2% | 2 | 23.700 | 5.3% | 4 | 46.188 | 7.6% | 3 | 35.205 | 2.0% | 4 | 46.940 | 0.9% | 4.7% |
| 0.0035664 | 4 | 46.123 | 3.9% | 4 | 46.232 | 5.1% | 2 | 23.667 | 5.2% | 4 | 46.123 | 7.8% | 3 | 35.156 | 1.9% | 4 | 46.875 | 0.8% | 4.7% |
| 0.0035614 | 4 | 46.059 | 4.0% | 4 | 46.167 | 4.9% | 2 | 23.634 | 5.0% | 4 | 46.059 | 7.9% | 3 | 35.107 | 1.8% | 4 | 46.809 | 0.7% | 4.7% |
| 0.0017214 | 9 | 50.090 | 4.4% | 8 | 44.630 | 1.4% | 4 | 22.847 | 1.5% | 9 | 50.090 | 0.2% | 6 | 33.938 | 1.6% | 8 | 45.250 | 2.7% | 2.4% |
| 0.0017164 | 9 | 49.945 | 4.1% | 8 | 44.500 | 1.1% | 4 | 22.781 | 1.2% | 9 | 49.945 | 0.1% | 6 | 33.839 | 1.9% | 8 | 45.119 | 3.0% | 2.3% |
| 0.0017114 | 9 | 49.799 | 3.7% | 8 | 44.371 | 0.8% | 4 | 22.714 | 1.0% | 9 | 49.799 | 0.4% | 6 | 33.740 | 2.2% | 8 | 44.987 | 3.3% | 2.3% |
| 0.0017064 | 9 | 49.654 | 3.4% | 8 | 44.241 | 0.5% | 4 | 22.648 | 0.7% | 9 | 49.654 | 0.7% | 6 | 33.642 | 2.5% | 8 | 44.856 | 3.5% | 2.3% |
| 0.0017014 | 9 | 49.508 | 3.1% | 8 | 44.111 | 0.3% | 4 | 22.582 | 0.4% | 9 | 49.508 | 1.0% | 6 | 33.543 | 2.8% | 8 | 44.724 | 3.8% | 2.4% |
| 0.0006264 | 24 | 48.606 | 1.3% | 22 | 44.661 | 1.5% | 11 | 22.863 | 1.6% | 25 | 50.632 | 1.3% | 17 | 34.990 | 1.4% | 23 | 47.340 | 1.8% | 1.5% |
| 0.0006214 | 24 | 48.218 | 0.5% | 22 | 44.304 | 0.7% | 11 | 22.680 | 0.8% | 25 | 50.227 | 0.5% | 17 | 34.711 | 0.6% | 23 | 46.962 | 1.0% | 0.7% |
| 0.0006164 | 24 | 47.830 | 0.4% | 22 | 43.948 | 0.1% | 11 | 22.498 | 0.0% | 25 | 49.823 | 0.4% | 17 | 34.432 | 0.2% | 23 | 46.584 | 0.2% | 0.2% |
| 0.0006114 | 24 | 47.442 | 1.2% | 22 | 43.591 | 0.9% | 11 | 22.315 | 0.8% | 25 | 49.419 | 1.2% | 17 | 34.152 | 1.0% | 23 | 46.206 | 0.6% | 1.0% |
| 0.0006064 | 24 | 47.054 | 2.0% | 22 | 43.235 | 1.7% | 11 | 22.133 | 1.6% | 26 | 50.976 | 2.0% | 17 | 33.873 | 1.8% | 23 | 45.828 | 1.4% | 1.8% |

Table 2. Time Step, *dt*, VS Error in Modeling Length of "Long" Pipes

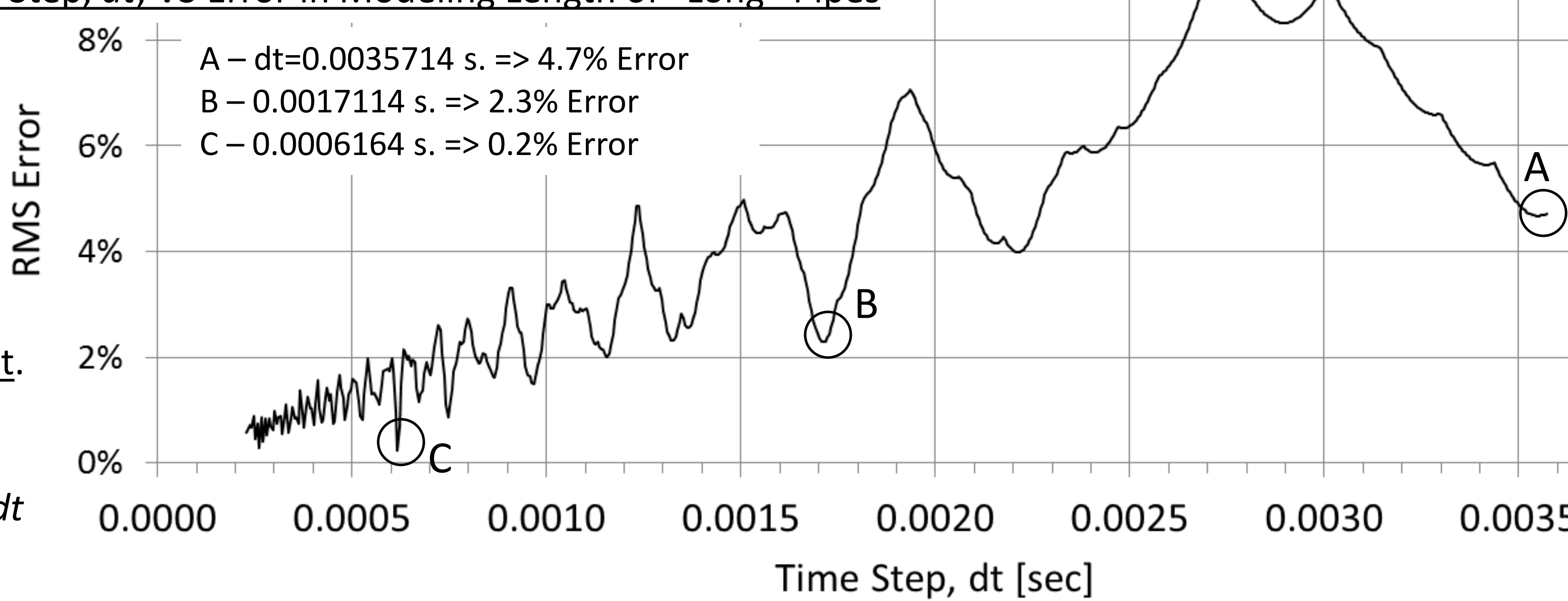



Table 2-Inset. RMS Error Vs Time Step, *dt*

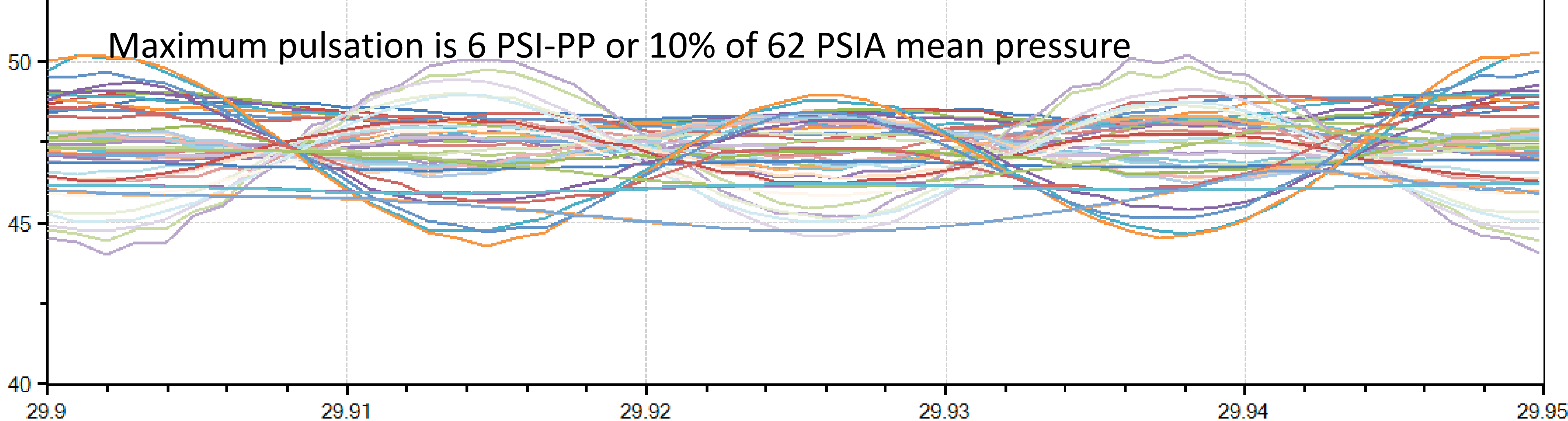
*Basic Data for Fig 1 Piping Network and Simulation Step Size, dt*

| | | | |
|---|---|---|---|
| *dt* | 0.0017114 | [sec.] | desired time step in the waterhammer software |
| *ρ* | 0.87 | [g/cc] | liquid density |
| μ | 0.95 | [CP] | viscosity |
| *B* | 140,000 | [psi] | liquid bulk modulus |
| *E* | 2.83E+07 | [psi] | pipe material elastic modulus |

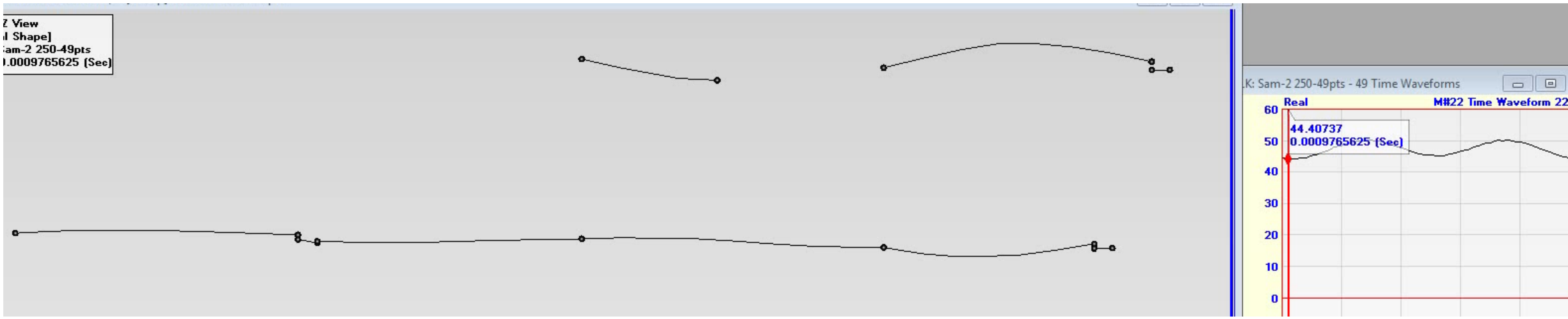
* - Enter wavespeed, *a* , directly into waterhammer software for volumes such as Dampener

| *Basic Data for each Pipe from Fig 1* | | | | *WH Software* | *Eq 3* | *Eq 4* | *Integer L/Li* | *n*Li* | *Error L' vs L* |
|---|---|---|---|---|---|---|---|---|---|
| col. 1 | col. 2 | col. 3 | col. 4 | col. 5 | col. 6 | col. 7 | col. 8 | col. 9 | col. 10 |
| *Pipe i* | D [in] | t [in] | L [ft] | c1 | a [ft/s] | Li [ft] | n [-] | L' [ft] | error |
| 1 | 3.26 | 0.12 | 48 | 1.06 | 3,233.2 | 5.533 | 9 | 49.799 | 3.7% |
| 2 | 1.682 | 0.109 | 3.2 | 1.06 | 3,323.9 | 5.689 | 1 | 5.689 | 77.8% |
| 3 | 3.26 | 0.12 | 44 | 1.02 | 3,240.8 | 5.546 | 8 | 44.371 | 0.8% |
| 4 | 1.682 | 0.109 | 22.5 | 1.11 | 3,318.1 | 5.679 | 4 | 22.714 | 1.0% |
| 5 | 3.26 | 0.12 | 50 | 1.06 | 3,233.2 | 5.533 | 9 | 49.799 | 0.4% |
| 6 | 2.157 | 0.109 | 34.5 | 1.084 | 3,285.9 | 5.623 | 6 | 33.740 | 2.2% |
| 7 | 2.157 | 0.109 | 46.5 | 1.084 | 3,285.9 | 5.623 | 8 | 44.987 | 3.3% |
| 8 | 12.191 | 0.188 | 3 | * | 2,890.9 | 4.947 | 1 | 4.947 | 64.9% |
| 9 | 12.191 | 0.188 | 3 | * | 2,890.9 | 4.947 | 1 | 4.947 | 64.9% |

TABLE 1b. Details of Waterhammer Modeling Sample Problem - *dt* = 0.0017114 sec



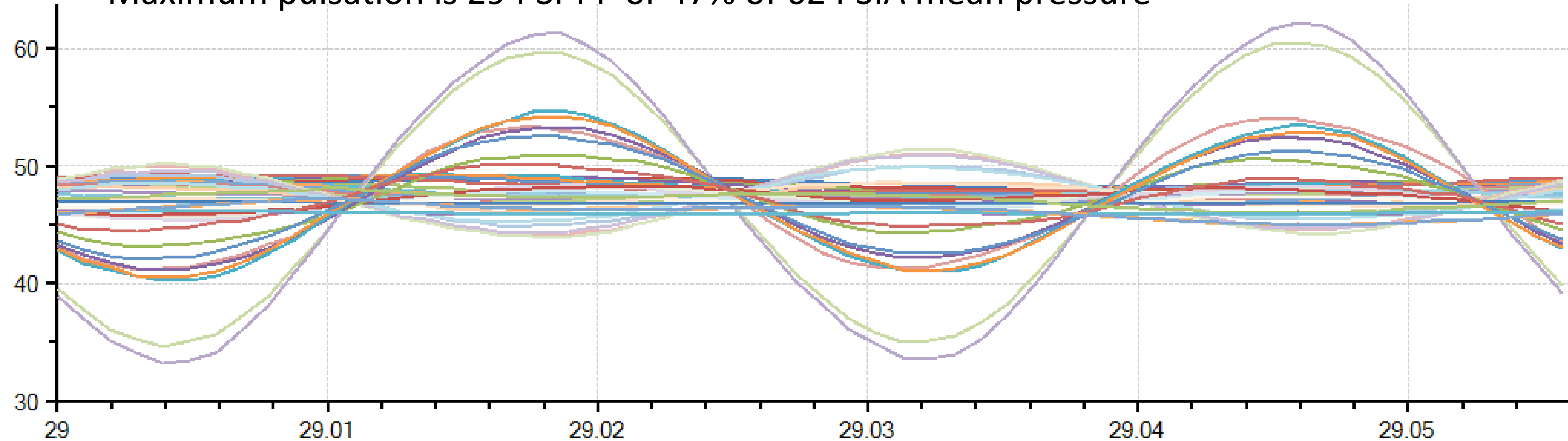

a) Superimposed Pressure Responses [PSIG] for 49 Points across Fig 1 System



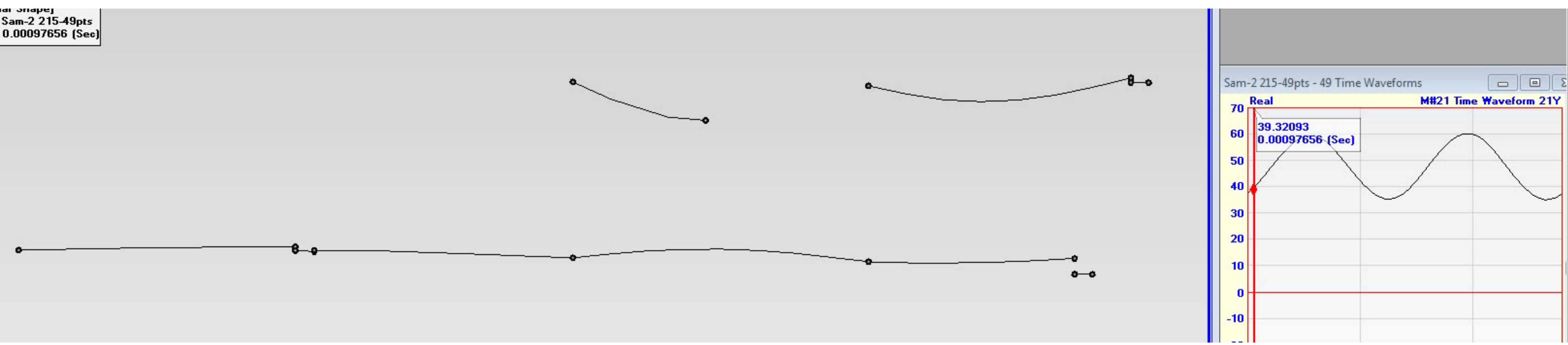

b) Animation of Operating Forced Response (CLICK Attach. “Fig 4b_Pr Anim 250 RPM.wmv”)

Fig 4. Pulsation Analysis Results for Fig 1 System with West Pump at 250 RPM

Maximum pulsation is 29 PSI-PP or 47% of 62 PSIA mean pressure

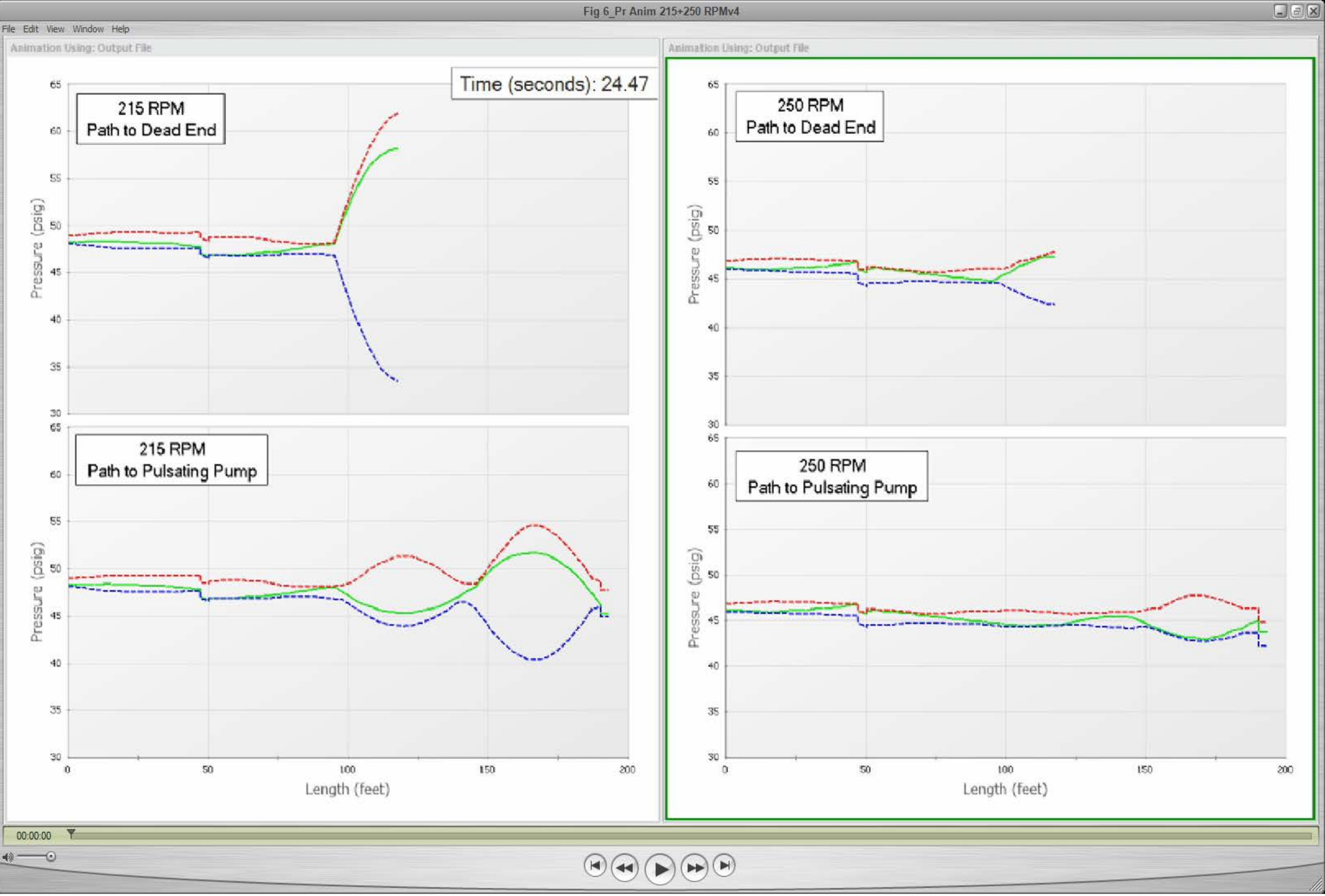



a) Superimposed Pressure Responses [PSIG] for 49 Points across Fig 1 System




b) Animation of Operating Forced Response (CLICK Attach. “Fig 5b_Pr Anim 215 RPM.wmv”)

Fig 5. Pulsation Analysis Results for Fig 1 System with West Pump at 215 RPM




CLICK Attachment "Fig 6 Pr Anim 215+250 RPM.mp4

Fig 6. Pulsation Animations – Transfer Pump to West PD Pump and Dead Leg Branch

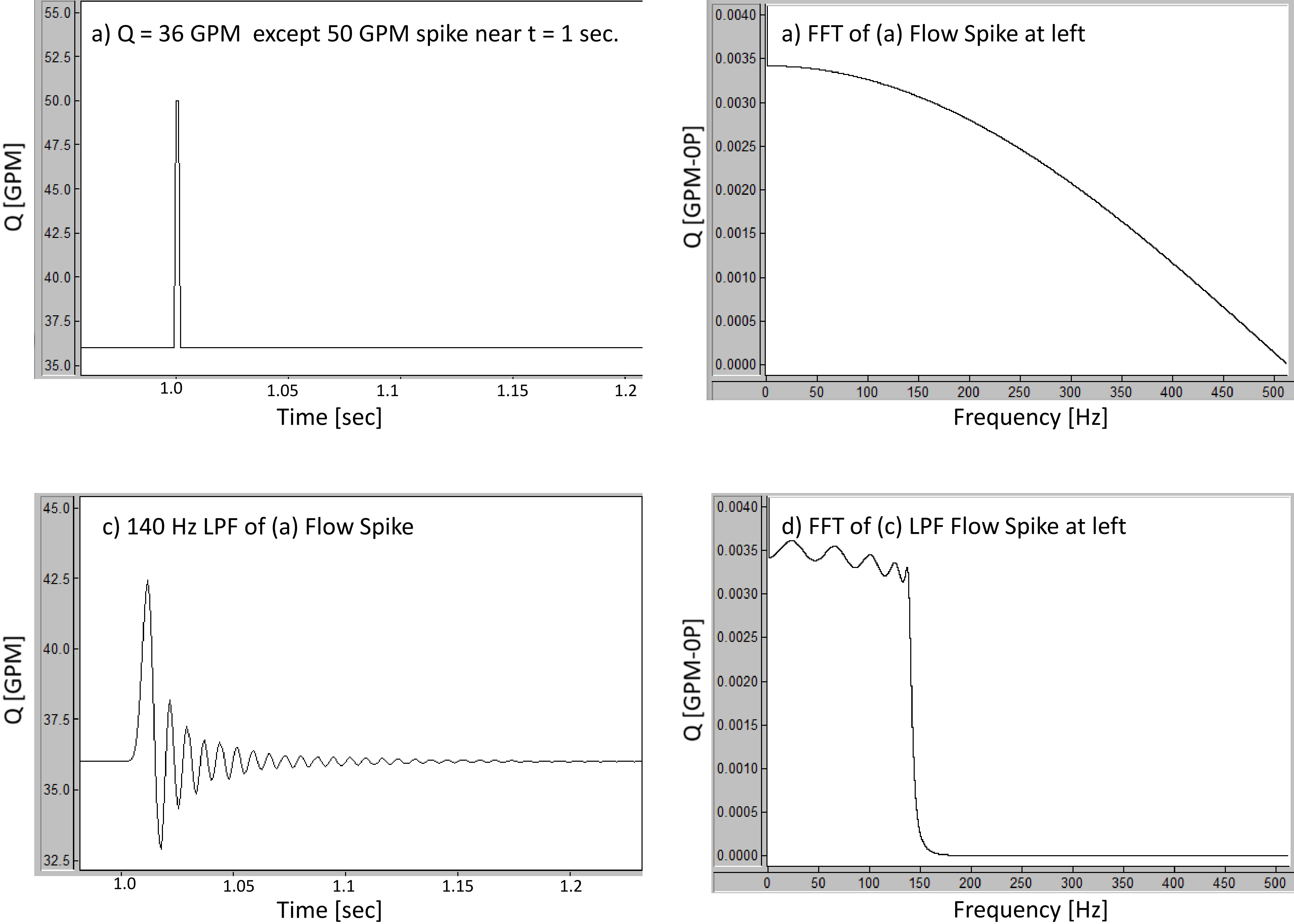



Fig 7. Simulated Flow Spike “Impact” Event at loc. “*j*” (West pump, J8)

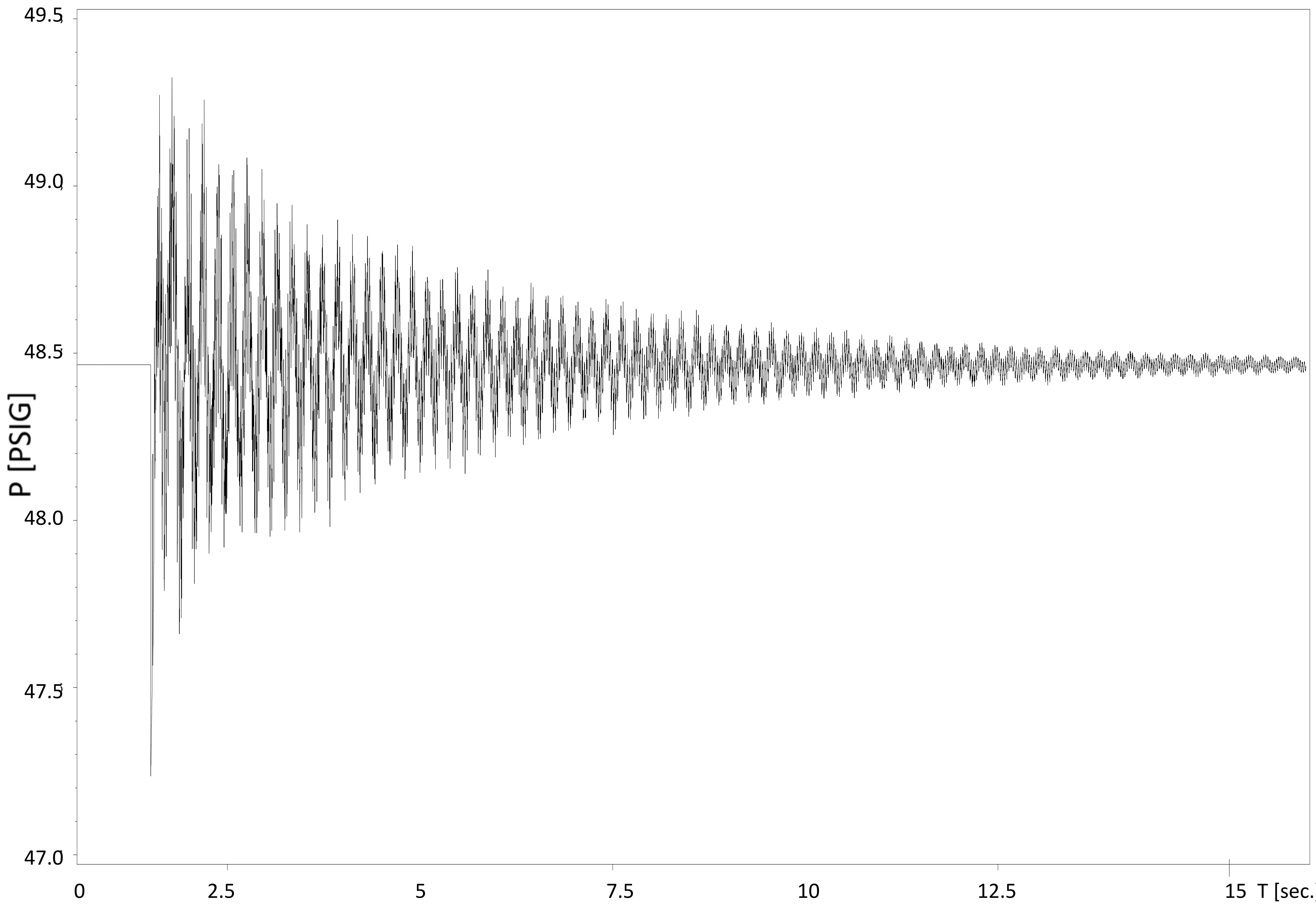



Fig 8. Simulated Pressure Response [PSI] at a loc. "*i*" to Fig 7 Impact Event

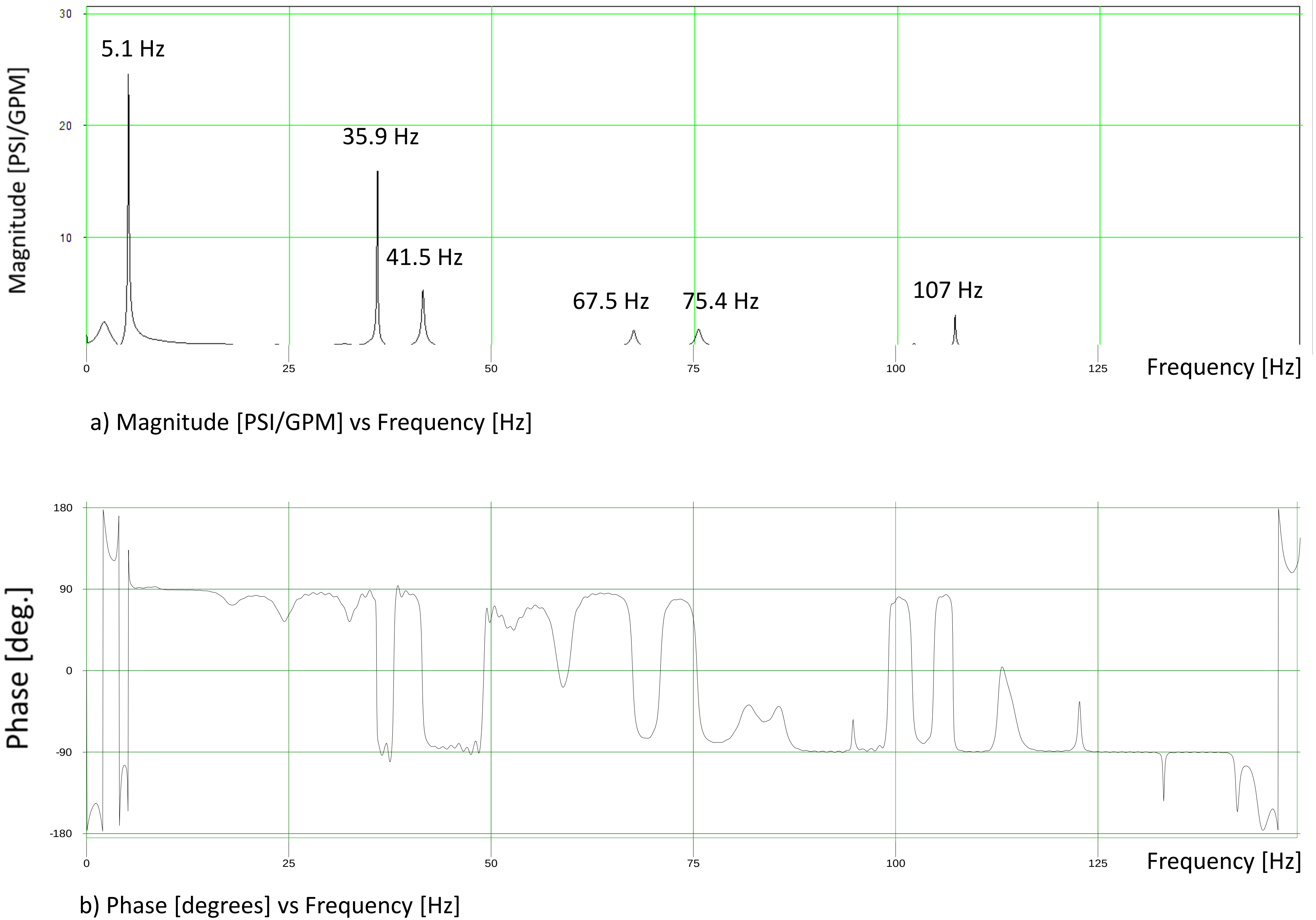
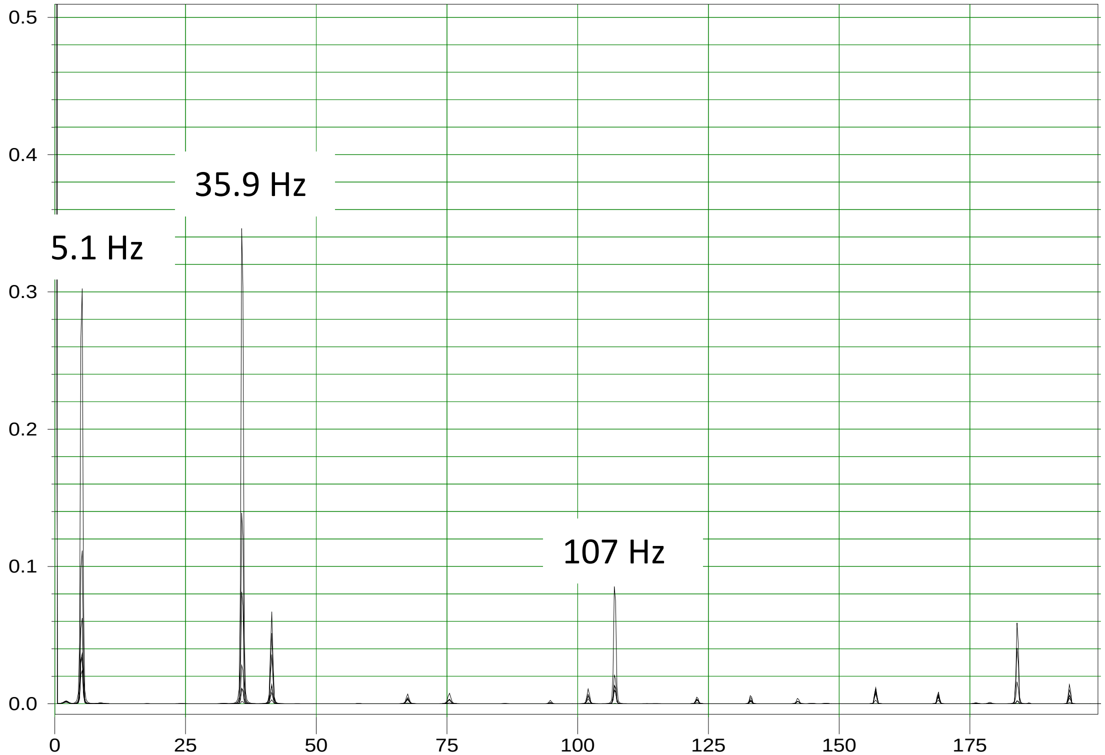



Fig 9. Bode Plot of an FRF Component – $h_{ij}(f) = P_i(f)/Q_j(f)$




a) Dynamic Pressures [PSI-Pk] at Several Locations Superimposed

| | Multiple of Run Speed | | | | | |
|---|---|---|---|---|---|---|
| f [Hz] | 5 | 10 | 15 | 20 | 25 | 30 |
| 5.13 | | | | | | |
| 35.88 | | 215.25 | 143.5 | | | |
| 41.50 | | 249 | 166 | | | |
| 67.47 | | | 269.88 | 202.41 | 161.93 | 134.94 |
| 75.44 | | | | 226.31 | 181.05 | 150.88 |
| 107.13 | | | | | 257.1 | 214.25 |

b) Run Speeds [RPM] Aligning with Important fn's

c) Animations of Mode Shapes (CLICK Attachment “Fig 10 fn Modes.wmv)

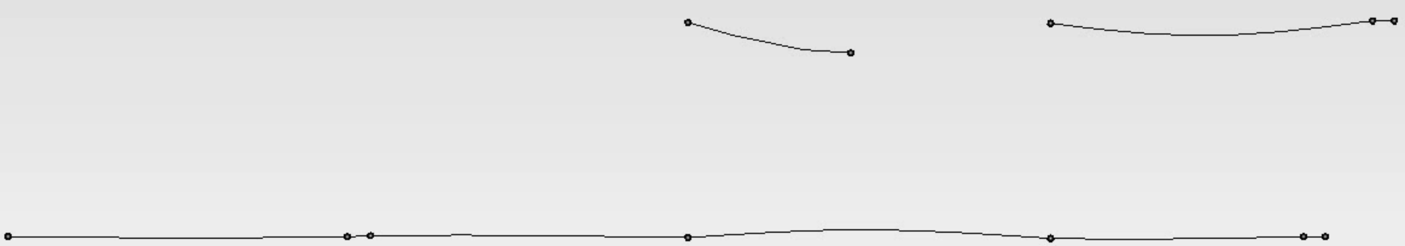
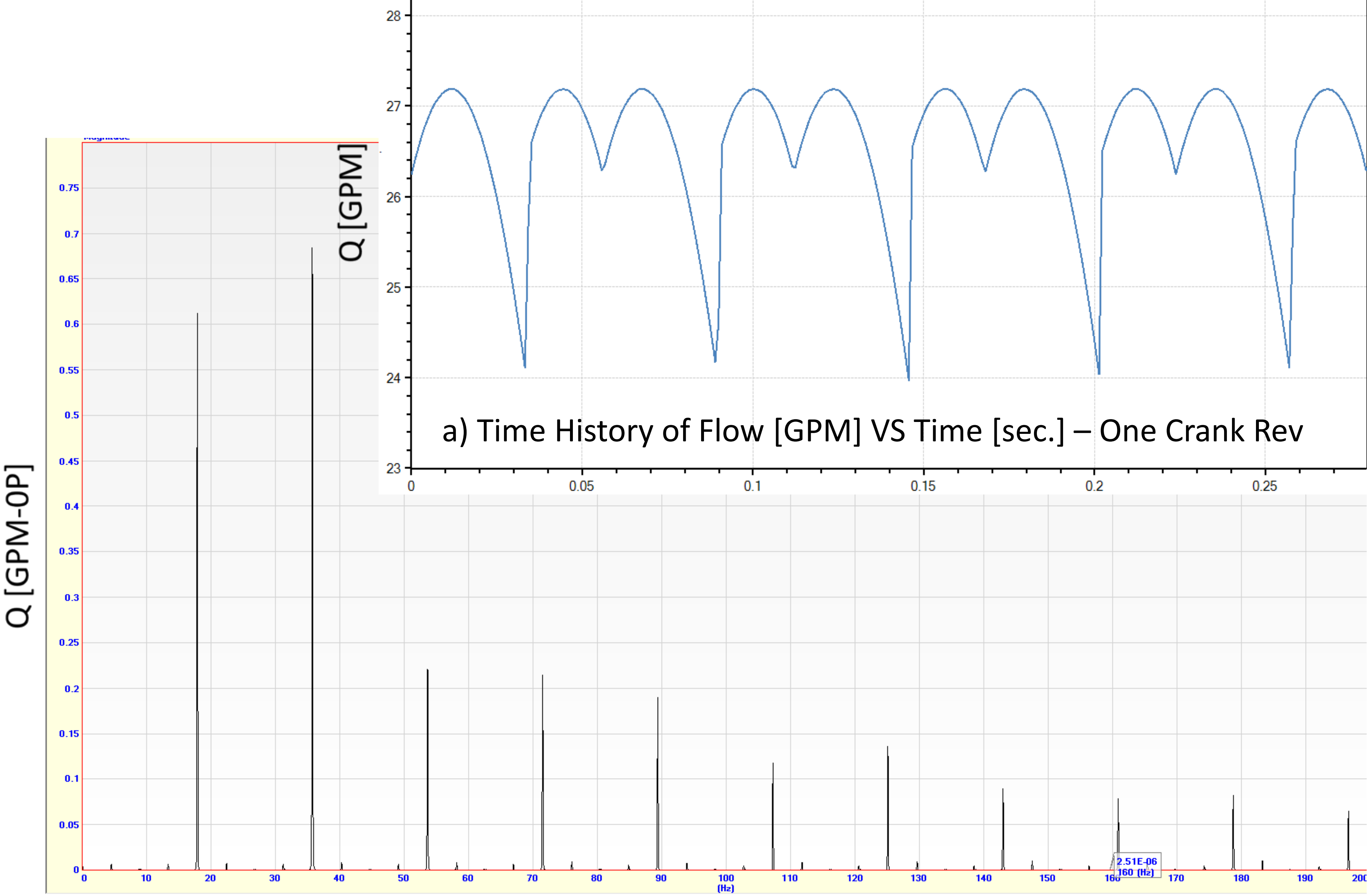


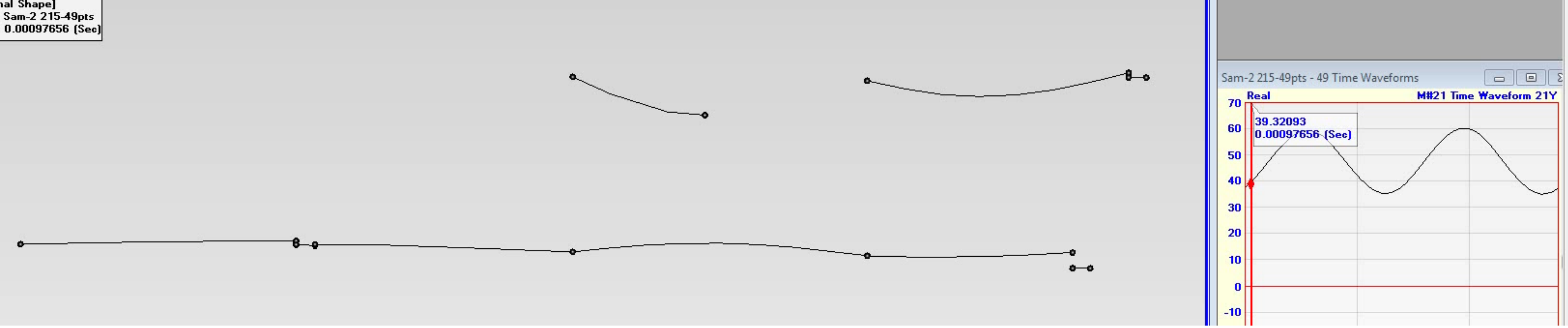

Fig 10. Natural Frequencies and Mode Shapes – based on FFT of 4 seconds following Fig 7 Impact



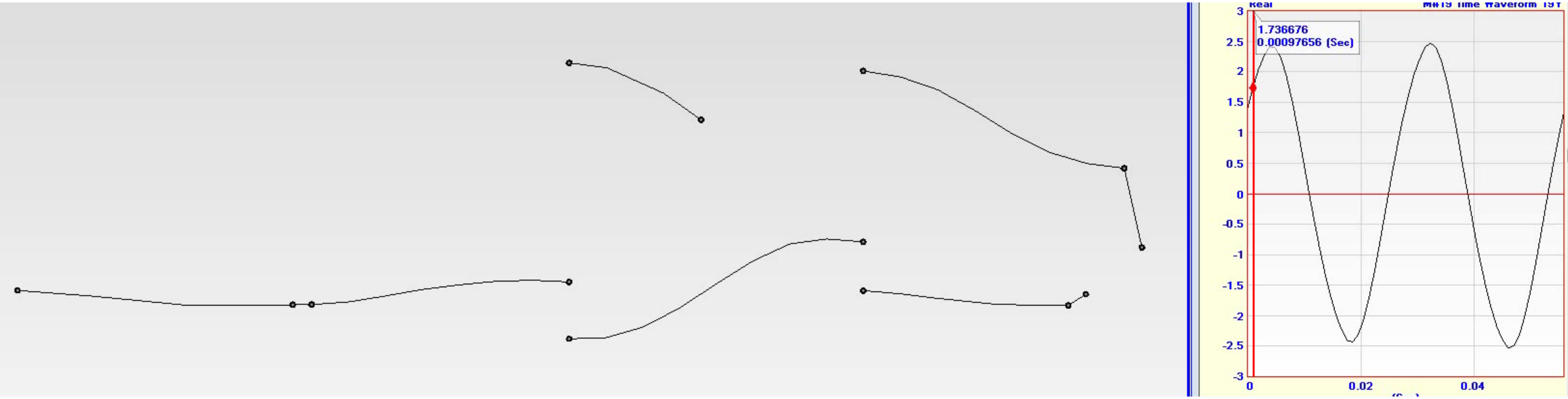
a) Time History of Flow [GPM] VS Time [sec.] – One Crank Rev

b) Frequency Content of Flow [GPM-0P] VS Frequency [Hz] (FFT of above figure 'a')

<u>Fig 11. Flow Forcing for the West Pump Running at 215.25 RPM</u>




a) PRESSURE Pulsation Response [PSI] (CLICK Attachment “Fig 5b_Pr Anim 215 RPM.wmv”)




b) FLOW Pulsation Response [GPM] (CLICK Attachment “Fig 12_Flow 215 RPM.wmv”)

<u>Fig 12. Pulsation Response ANIMATIONS for West Pump Running at 215.25 RPM</u>

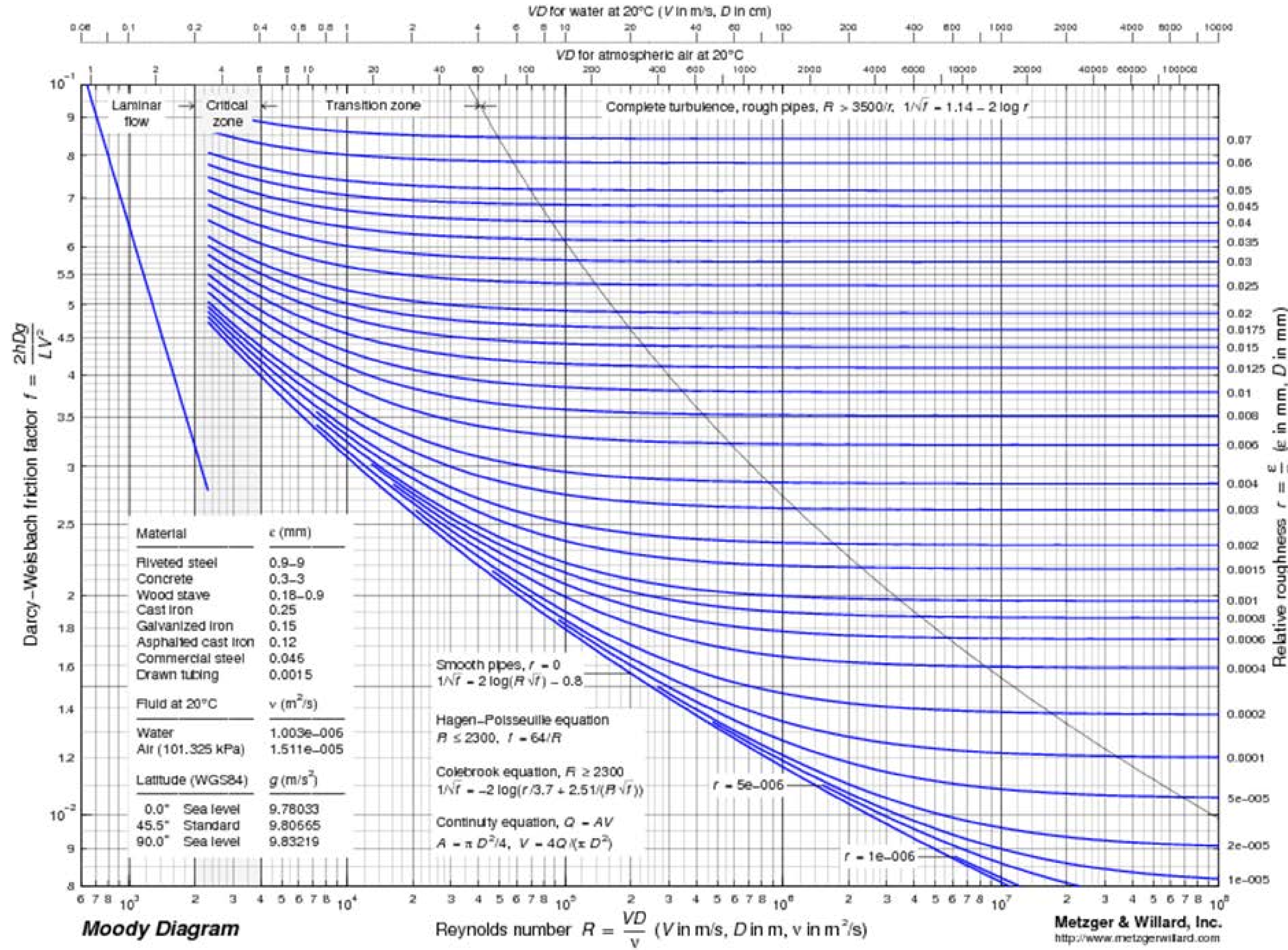



Fig 13. Moody Diagram - Friction Factor as Function of Reynolds Number and Pipe Roughness

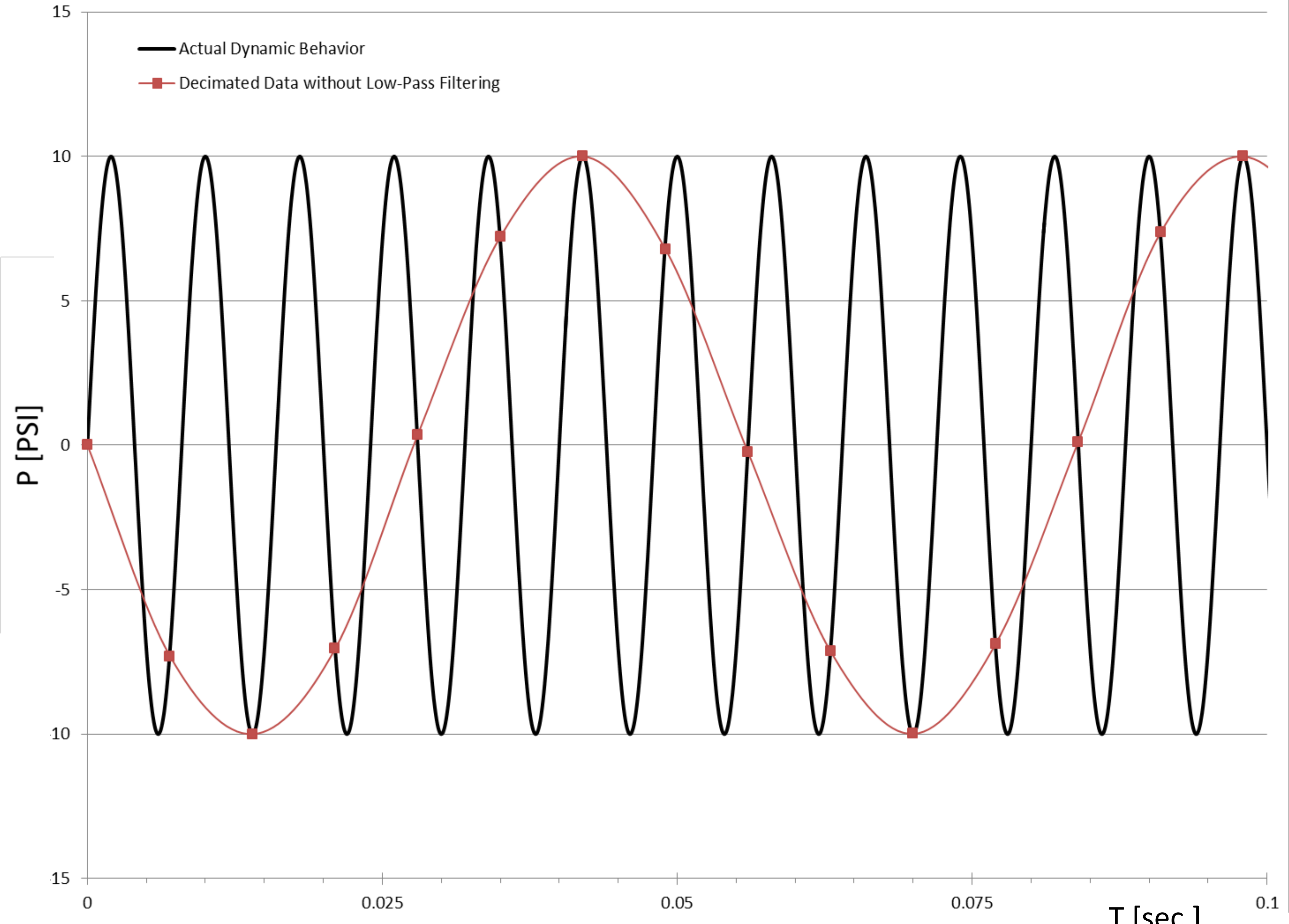



Fig 14.  Illustration of “Signal Aliasing”

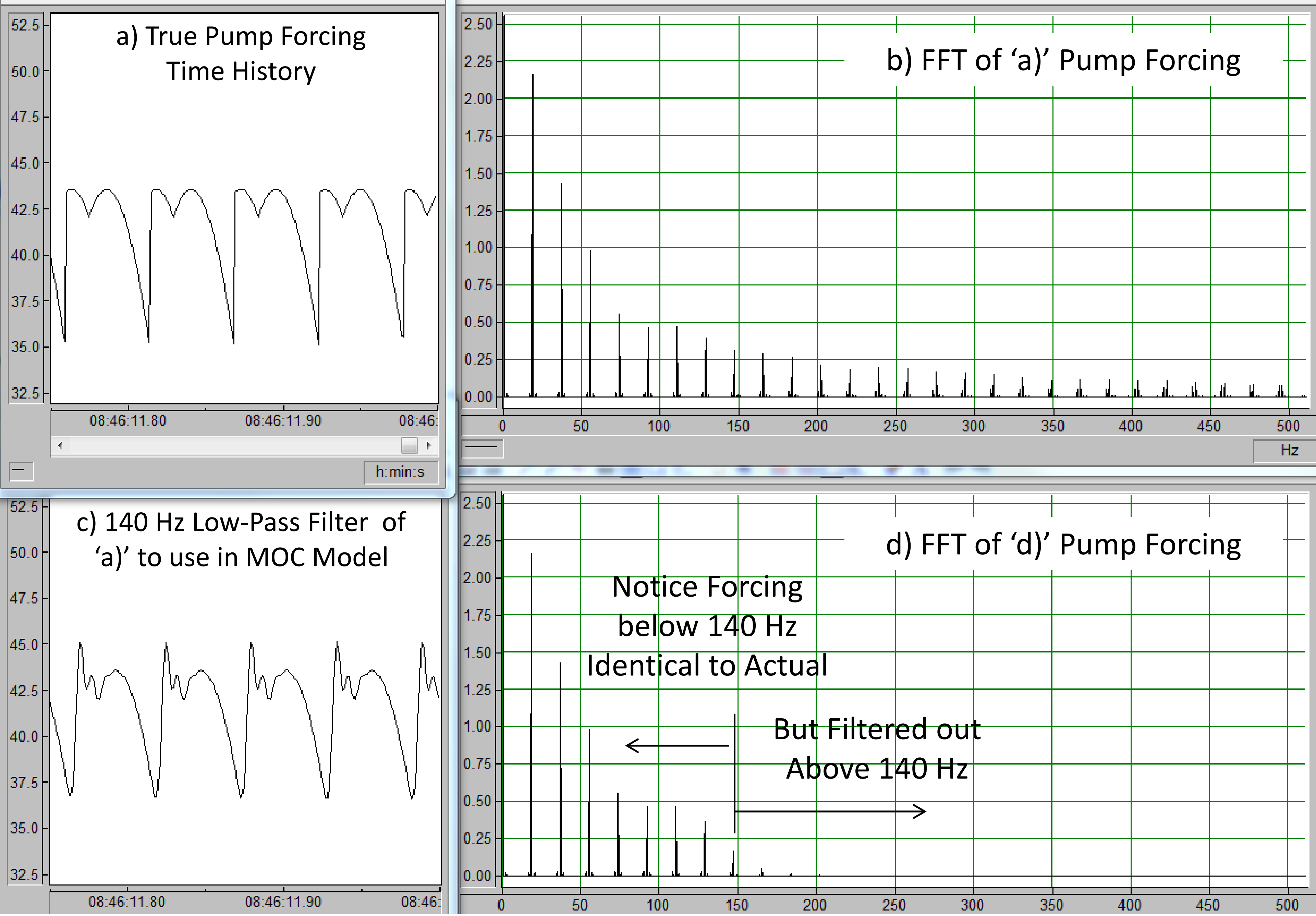



Fig 15. PD Pump Flow Forcing [GPM] to Achieve Desired Responses Only in 0–140 Hz Range