Applied Flow Technology

Modeling Choked Flow Through an Orifice

Summary

Periodically AFT Arrow users pose the question of what CdA value to use for modeling choked flow through an orifice. This paper reviews the subject to provide guidance in this area. As will be seen, there is not one answer.

Discussion

AFT Arrow provides several options in modeling an orifice, perhaps the two most commonly used being the Sharp-Edged and Cd (discharge coefficient) orifice models.

The Sharp-Edge model uses Idelchik's 'sharp-edged orifice' equation found on page 221 of reference (1);

$$k_1 = \frac{\Delta p}{\rho w_1^2 / 2} = \left[\left(1 - \frac{F_0}{F_1}\right) + 0.707 \left(1 - \frac{F_0}{F_1}\right)^{0.5} \right]^2 \left(\frac{F_1}{F_0}\right)^2$$

Where:

w_1 = velocity in upstream pipe

F0, F1 = orifice and upstream areas, respectively

Idelchik lists this equation as valid for an orifice thickness to hydraulic diameter ratio of 0 to 0.015. Notably, the above relationship is for incompressible flow[1] (as are all resistance coefficients in Idelchik).

When a Cd is specified, AFT Arrow uses the following equation from page 339 of reference (9).

$$\dot{m} = C_d A P_{down} \sqrt{\frac{2}{RT}\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{P_{up}}{P_{down}}\right)^{\frac{\gamma-1}{\gamma}} \left(1 + \left(\frac{P_{down}}{P_{up}}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

[1] Ref: Idelchik Page 4 footnote.

AFT Arrow evaluates sonic flow if a C_dA is specified based on equation 3.23 of Saad;

$$\frac{\dot{m}}{A} = \frac{p_0}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}}\frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}}$$

For choked flow, M = 1 and A = C_dA (the same relationship is used for endpoint and expansion choking with A equaling the physical area of the pipe discharge at the endpoint or entering the expansion).

Crane

For high Reynolds numbers from about 10^5 and higher, Crane, page A-20, indicates an essentially constant flow coefficient for sharp edged orifices that depends only on the orifice to pipe diameter ratio. C_d is related to the flow coefficient C by the following:

$$C_d = C\sqrt{1-\beta^4}$$

Where β = orifice diameter / upstream pipe diameter.

While C increases with increasing β to a value of ~0.74 for β = 0.75, because of the above relationship C_d is ~0.6 irrespective of β and this value is often used in calculating C_dA by multiplying the physical orifice area times 0.6.

In a related discussion on page 2-14 of ref. (3), the following equation is provided by Crane for the flow of gases and vapors:

$$q = YCA\sqrt{\frac{2g(144)\Delta P}{\rho}}$$

Where q is volumetric flow.

This is the same form of equation used for incompressible flow with the addition of the expansion factor Y, which relates the density of the fluid at the orifice to the upstream density. Further, Crane states;

> *"When the absolute inlet pressure is greater than this amount, flow through nozzles should be calculated as outlined on the following page."*

...which goes on to state;

> *"A smoothly convergent nozzle has the property of being able to deliver a compressible fluid up to the velocity of sound in its minimum cross section or throat."*

Values of Y as a function of the pressure drop ratio (defined as the pressure differential divided by the upstream pressure) and the expansion coefficient k, are provided on page A-21. For nozzles, Y has a minimum value corresponding to choked flow. No such minimum value of Y is provided for a square edge orifice indicating the limitations in using the Crane method to evaluate choked flow through an orifice.

Cunningham

In his paper *Orifice Meters With Supercritical Compressible Flow*, ref. (4), Cunningham states;

> "For a well-formed convergent nozzle, the (experimental) maximum flow ratio is essentially identical with the (theoretical) critical-pressure ratio. Evidently the occurrence of sonic velocity at the throat of the nozzle prevents flow response to changes in the discharge pressure.
>
> Contrary to the behavior of the convergent nozzle, the square-edged orifice does not exhibit a maximum flow ratio. Rather, experiment shows that the flow rate (for constant upstream conditions) continues to increase at all pressure ratios between the critical and zero; this range is defined as the 'supercritical" range of ratios"

Cunningham goes on to provide a brief description of a previous investigation by Stanton, explaining that shock disturbances were evident at all orifice pressure ratios below critical, but that the location of the critical pressure was downstream of the orifice and moves toward the orifice as the discharge pressure is lowered.

The bulk of Cunningham's paper reports on the empirically determined expansion coefficient to be used in the "ASME Equation";

$$G = KYA_0\sqrt{2g_c\rho_1\Delta p}$$

Two sets of equations for the expansion coefficient are developed. One for "flange taps" and another for "pipe taps". Each set consists of two equations, one for high pressure ratios and the other for lower pressure ratios as follows (r = P_2/P_1 or $P_{downstream}/P_{upstream}$);

Pipe taps -

r >= 0.77 $$1-\left(0.333+1.145\left(\beta^2+0.7\beta^5+12\beta^{13}\right)\right)\frac{1-r}{\kappa}$$

r < 0.77 $$Y_{0.77}-0.364(0.77-r)$$

Flange taps -

r >= 0.63 $$1.0-\left(0.41+.035\beta^4\right)\frac{1-r}{\kappa}$$

r < 0.63 $$Y_{0.63-0.3501(0.63-r)}$$

Driskell

In a discussion of flow through nozzles, restrictions and enlargements at www.isa.org, ref. (5), reference is made to L.R. Driskell as follows;

> "Driskell repeated his stand in a later publication, Control-Valve Selection and Sizing. He described choked flow as being that condition where flow cannot be increased by lowering downstream pressure. He further described the vena contracta as migrating upstream to coincide with the orifice when the flow is completely 'choked' (meaning it has reached its maximum steady-state value of flow)."

This appears to be both similar to and contradict Cunningham, for while a description of a moving vena contracta based on the downstream pressure is provided, it also states the vena contracta 'coincides' with the orifice when it is 'choked'. Contrary to this, Stanton's data (included in Cunningham's paper), clearly shows that while the vena contracta approaches the orifice with decreasing downstream pressure it never reaches it. Further, if the vena contracta 'coincides' with the orifice as Driskell asserts, this would imply a C_d equal to 1.0, a seeming contradiction to accepted values of C_d less than 1.0 for 'well formed' nozzles.

ENGSoft

That an orifice does not exhibit a maximum flow at a critical pressure ratio is reflected in other sources as well. For example, ENGSoft Inc. which states on their website www.engsoft.co.kr, ref. (6);

> "In case of orifice, actually there is no critical pressure. The mass flow rate increases as much as the discharge pressure is decreased till zero absolute pressure."

Ward-Smith

Optimal Systems, www.optimal-systems.demon.co.uk, ref. (7), provides a description of a series of experiments conducted by Ward-Smith wherein it is stated;

> "In summary, the principal parameter affecting sonic discharge coefficient is the aspect ratio of the orifice, expressed as the ratio of the plate thickness to the orifice diameter (t/d)...
>
> Once sonic velocity has been achieved, further reduction of the downstream pressure cannot further increase the velocity through the vena contracta, but if the orifice plate is thin, it can increase the vena contracta's size.
>
> Further reductions in downstream pressure cause the vena contracta to move upstream and to consequently increase in area. Ultimately, at high pressure ratios, the vena contracta can reach the upstream edge of the orifice, when its area would equal that of the orifice and the discharge coefficient would be unity."

Like Driskell, the vena contracta is described as reaching the orifice at 'high' pressure ratios (indeed, the upstream edge of the orifice) yielding a discharge coefficient of 1.0.

Ward-Smith is further quoted on this subject in http://www.eng-tips.com/viewthread.cfm?qid=106580&page=1 as providing the following choked flow Cd values:

sharp edge, t/d= 0, Cd = 1.0

thin plate (0<t/d<1)Cd varies smoothly from 1 to 0.81 as function of t/d.

thick plate (1<t/d<7) Cd = 0.81 constant

very thick plate (t/d > 7) Cd less than 0.81 per Fanno friction

Where t/d equals the ratio of the orifice thickness to its diameter.

ASME

Finally, we have the current ASME standard MFC-3M-1989. It is worthwhile to review the scope of application of this standard as it pertains to this discussion;

- Applies only to pressure difference devices in which the flow remains turbulent and subsonic.
- Within the pipe size and Reynolds number limits specified.
- Orifice to upstream pipe diameter ratio of 0.2 > 0.7.
- Ratio of downstream to upstream pressure >= 0.75.

Of particular note, the ASME standard is limited to pressure ratios greater than the critical pressure ratio and does not cover choked flow through an orifice.

Mass flow -

$$q_m = \frac{\pi}{4} C Y_1 d^2 \sqrt{\frac{2 \Delta p \rho_{f1}}{1-\beta^4}}$$

Equations for C -

For D and D/2 taps (upstream ID >= 2.3 in.)

$$C = 0.5959 + 0.0312\beta^{2.1} - 0.1840\beta^8 + 0.0390\beta^4 \left(1-\beta^4\right)^{-1} - 0.01584\beta^3 + 91.71\beta^{2.5} R_D^{-0.75}$$

For flange taps

upstream ID >= 2.3 in.

$$C = 0.5959 + 0.0312\beta^{2.1} - 0.1840\beta^8 + 0.0900D^{-1}\beta^4 \left(1-\beta^4\right)^{-1} - 0.0337D^{-1}\beta^3 + 91.71\beta^{2.5} R_D^{-0.75}$$

2 in. < upstream ID < 2.3 in.

$$C = 0.5959 + 0.0312\beta^{2.1} - 0.1840\beta^8 + 0.0390D^{-1}\beta^4 \left(1-\beta^4\right)^{-1} - 0.0337D^{-1}\beta^3 + 91.71\beta^{2.5} R_D^{-0.75}$$

Expansion factor, Y -

$$Y_1 = 1 - \left(0.41 + 0.35\beta^4\right)\Delta p / \left(\kappa p_1\right)$$

Note that this is the same equation as presented by Cunningham for pipe taps with a pressure ratio >= 0.63.

Comparative Results

Runs were made in AFT Arrow along with corresponding calculations using ASME MFC-3 (ignoring the pressure ratio limitation) and Cunningham. AFT Standard air is used as the fluid with the orifice upstream pressure constant at 50 psia @ 70F and downstream pressure varying from 45 psia to 5 psia. Inlet and outlet pipes have a 4" ID> Comparison was made to both AFT Arrow's 'Sharp-Edged' and Cd orifice models.

The following notes apply to each modeling/calculation method:

AFT Arrow "Sharp-Edged" varying CdA – Orifice specified with a 1" diameter with CdA calculated over a range of Cd values from 0.6 to 1.0 x the orifice area.

AFT Arrow Used Specified Cd – Variable Cd value 0.6 (very nearly equal to the ASME value for C of 0.5979865) to 0.9, CdA calculated as Cd x orifice area.

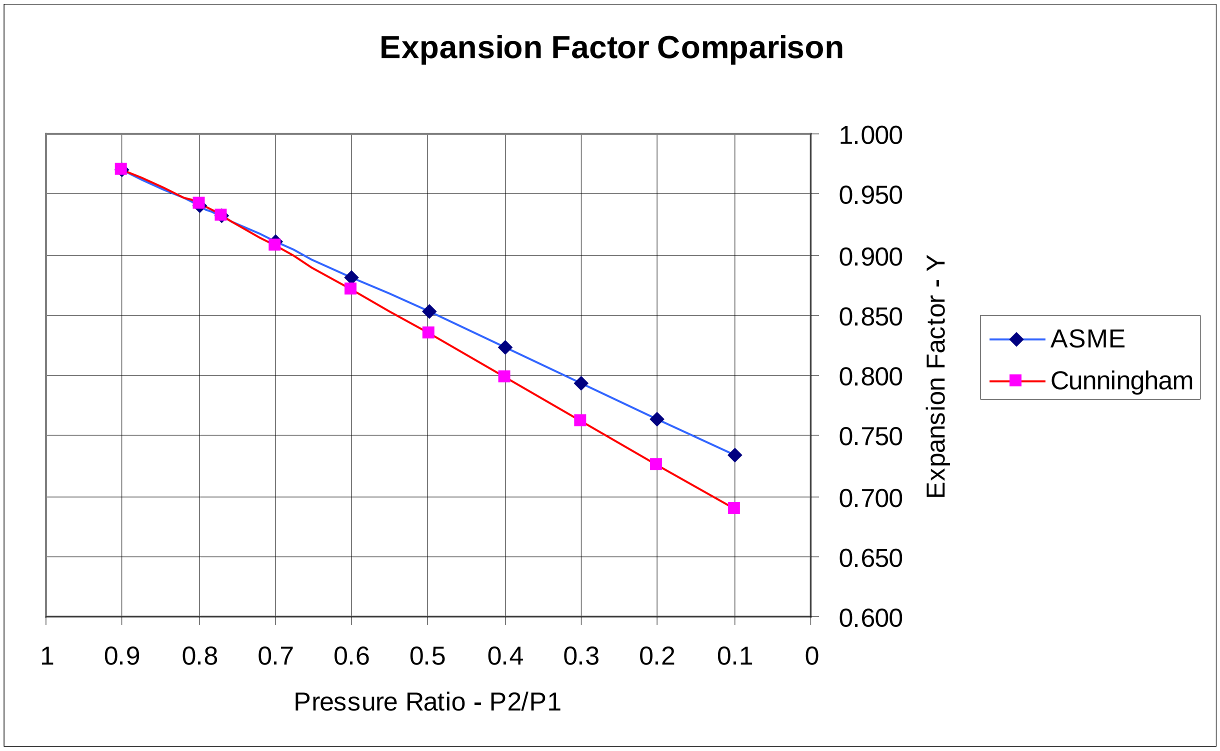
AFT Arrow Used Specified Cd – Variable Cd value 0.6 to 0.9, **no CdA specified.**

ASME MFC-3 - For ←p/p > 0.04, which includes all of the cases considered here, an arithmetic average k should be used, which was calculated using AFT Arrow's values for upstream and downstream gamma (in all cases, gamma average is equal to or very nearly 1.4). Expansion factor is calculated from the MFC-3 equation for Y, though this standard is limited to pressure ratios >= 0.75; i.e. a downstream pressure of 37.5psia.

Cunningham – Uses Cunningham's value of 0.6068 for K, the orifice discharge coefficient, applicable to pipe taps and a beta up to 0.4.

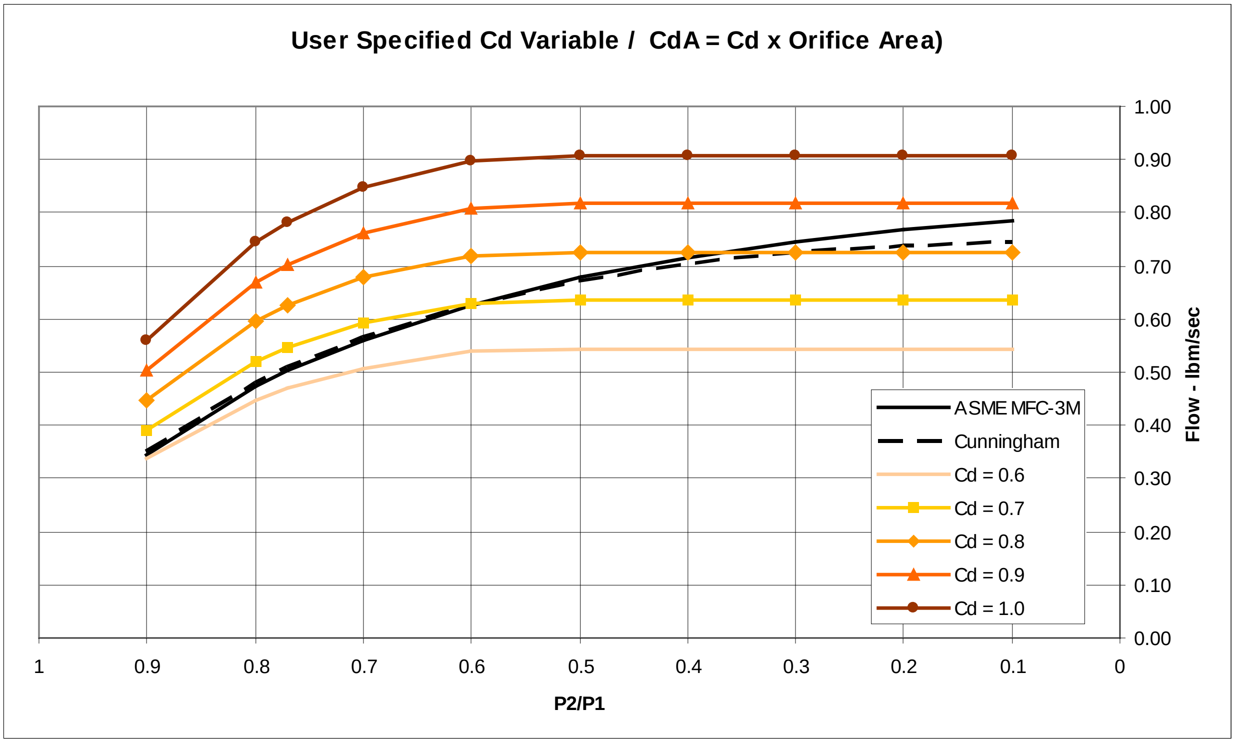
As they both use very similar discharge coefficient values, C = 0.5979865 for ASME and

K = 0.6068 for Cunningham, flows differ by the value of Y used, which compares as follows.

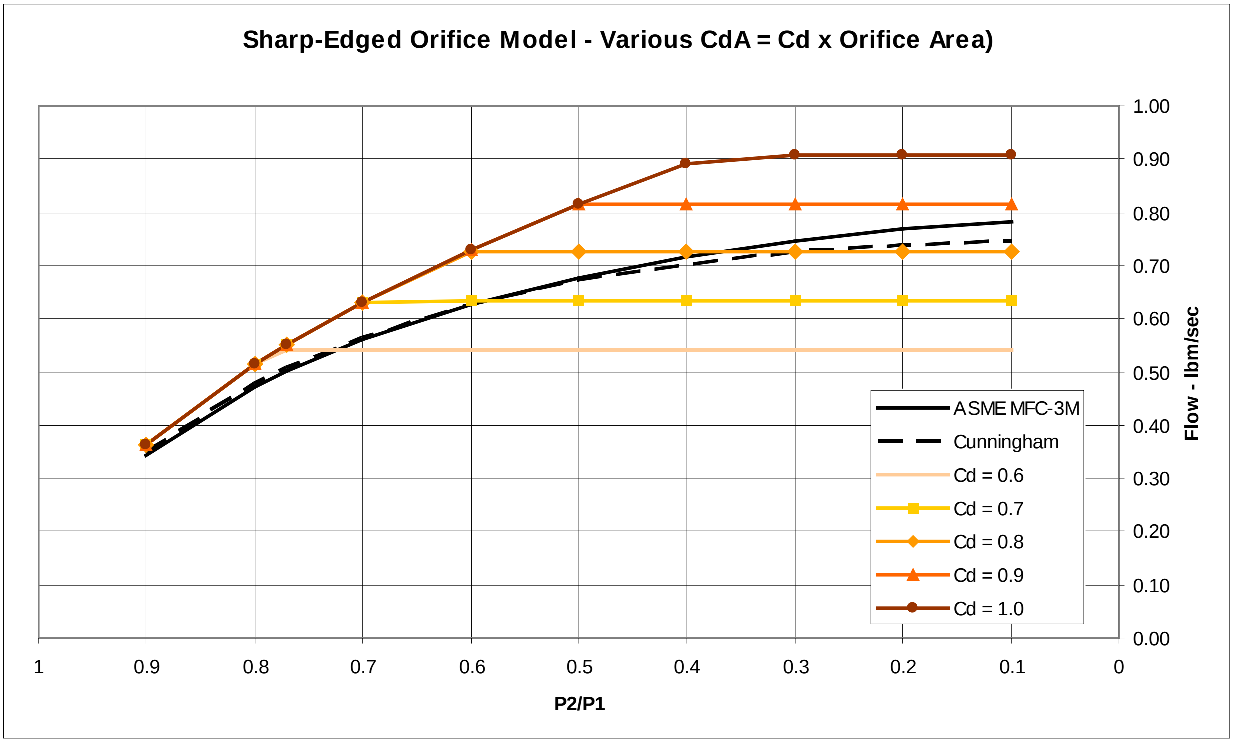


The following tables show flow in lbm/sec over a range of orifice pressure ratios for each of the above listed AFT Arrow orifice models and as calculated using ASME and Cunningham.

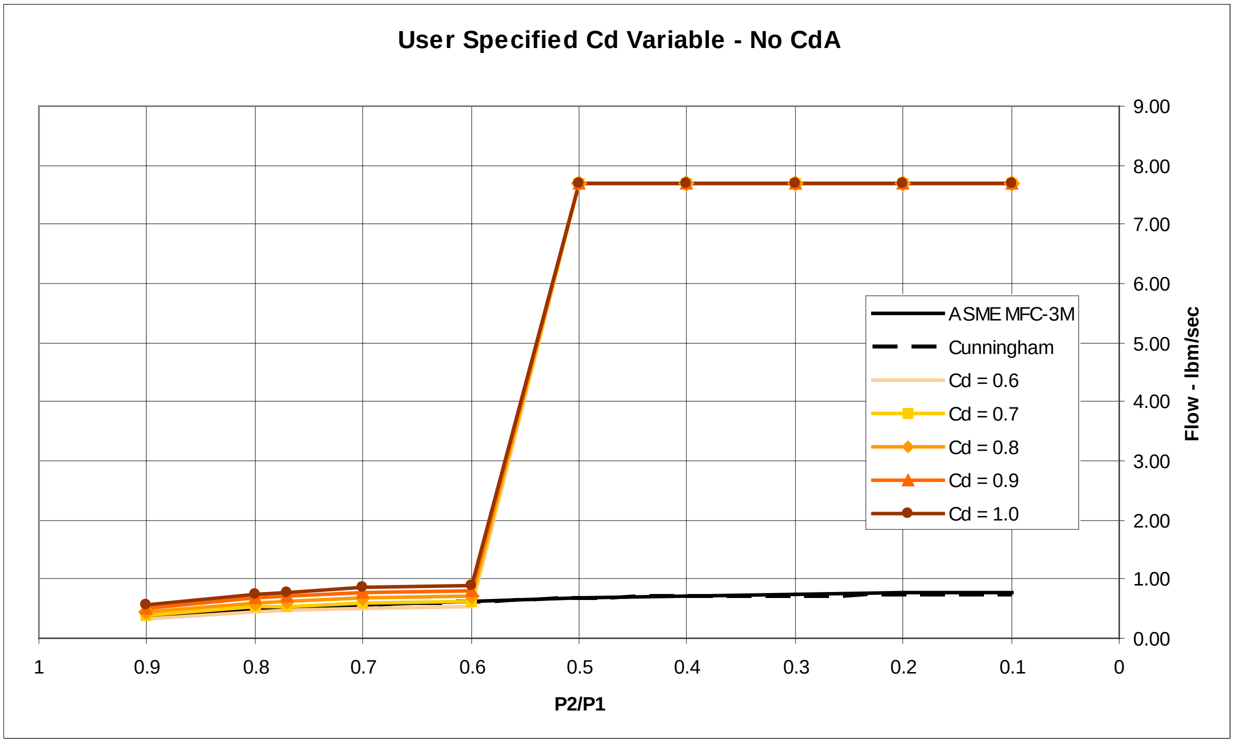
			User Specified Cd = 0.6 / Various CdA (Cd x A)				
P2/P1	ASME MFC-3M	Cunningham	Cd = 0.6	Cd = 0.7	Cd = 0.8	Cd = 0.9	Cd = 1.0
0.9	0.34	0.35	0.34	0.39	0.45	0.50	0.56
0.8	0.47	0.48	0.45	0.52	0.59	0.67	0.74
0.77	0.50	0.51	0.47	0.55	0.62	0.70	0.78
0.7	0.56	0.57	0.51	0.59	0.68	0.76	0.85
0.6	0.63	0.63	0.54	0.63	0.72	0.81	0.90
0.5	0.68	0.67	0.54	0.64	0.73	0.82	0.91
0.4	0.72	0.70	0.54	0.64	0.73	0.82	0.91
0.3	0.75	0.73	0.54	0.64	0.73	0.82	0.91
0.2	0.77	0.74	0.54	0.64	0.73	0.82	0.91
0.1	0.78	0.74	0.54	0.64	0.73	0.82	0.91



			Sharp-Edged - Cd for CdA				
P2/P1	ASME MFC-3M	Cunningham	Cd = 0.6	Cd = 0.7	Cd = 0.8	Cd = 0.9	Cd = 1.0
0.9	0.34	0.35	0.36	0.36	0.36	0.36	0.36
0.8	0.47	0.48	0.51	0.51	0.51	0.52	0.52
0.77	0.50	0.51	0.54	0.55	0.55	0.55	0.55
0.7	0.56	0.57	0.54	0.63	0.63	0.63	0.63
0.6	0.63	0.63	0.54	0.64	0.73	0.73	0.73
0.5	0.68	0.67	0.54	0.64	0.73	0.81	0.82
0.4	0.72	0.70	0.54	0.64	0.73	0.82	0.89
0.3	0.75	0.73	0.54	0.64	0.73	0.82	0.91
0.2	0.77	0.74	0.54	0.64	0.73	0.82	0.91
0.1	0.78	0.74	0.54	0.64	0.73	0.82	0.91



			User Specified Cd - No CdA				
P2/P1	ASME MFC-3M	Cunningham	Cd = 0.6	Cd = 0.7	Cd = 0.8	Cd = 0.9	Cd = 1.0
0.9	0.34	0.35	0.34	0.39	0.45	0.50	0.56
0.8	0.47	0.48	0.45	0.52	0.59	0.67	0.74
0.77	0.50	0.51	0.47	0.55	0.62	0.70	0.78
0.7	0.56	0.57	0.51	0.59	0.68	0.76	0.85
0.6	0.63	0.63	0.54	0.63	0.72	0.81	0.90
0.5	0.68	0.67	7.69	7.69	7.69	7.69	7.69
0.4	0.72	0.70	7.69	7.69	7.69	7.69	7.69
0.3	0.75	0.73	7.68	7.68	7.68	7.68	7.69
0.2	0.77	0.74	7.69	7.69	7.69	7.69	7.69
0.1	0.78	0.74	7.68	7.68	7.68	7.68	7.69



Observations & Conclusions

AFT Arrow does not currently use a compressible method to model subsonic flow in an orifice (since no recognized standard covers the full range of pressure ratios). While it faithfully reproduces Idelchik's loss values and the Cd method, both are incompressible methods.

ASME and Cunningham yield very similar results down to a pressure ratio of about 0.4, with ASME yielding an increasingly greater flow as pressure ratio progresses to lower values due mainly to the difference in the value of the expansion factor.

Using the ASME C value of ~0.6 as the C_d in AFT Arrow produces a flow rate similar to ASME and Cunningham at high pressure ratios. As pressure ratio is reduced the value of Cd must be increased to yield a comparable flow rate. Flow with the user specified Cd is limited at sufficiently low pressure ratios by choked flow, which is determined by the CdA. For example, at a pressure ratio of 0.6 the flow using the user specified Cd of 0.7 (and corresponding CdA of 0.7 x orifice area) matches that of ASME and Cunningham while at a pressure ratio of 0.1 a Cd of approximately 0.8 to 0.9 is needed to match.

Using AFT Arrow's sharp-edged orifice, which implements Idelchik's equation, flow match ASME and Cunningham at high pressure ratios, but deviates in sub-sonic flow at lower pressure ratios with the sharp-edged orifice model yielding an increasingly higher flow rate. The sharp-edge orifice model flow is limited by choking, the flow rate then a function of the CdA value specified. At a pressure ratio of 0.6, for example, a CdA = 0.7 x orifice area closely matches the ASME and Cunningham flow rates while at a pressure ratio of 0.1 a CdA of 0.8 to 0.9 is needed, just as with the user specified Cd orifice model.

As to what the various references reviewed say about choked flow in an orifice, they are inconclusive. Cunningham, and by reference Stanton, clearly indicate a flow limit does not occur at the critical pressure ratio. Notably, these are substantiated by empirical data.

Driskell says the vena contracta coincides with the orifice throat at sufficiently high pressure ratios, though what pressure ratio this corresponds to is not clear.

Ward-Smith describes sonic velocity occurring at the vena contracta, with further downstream pressure reduction causing the vena contracta to move upstream, eventually coinciding with the orifice throat and yielding a C_d of 1.0. Referring to either of the curves above with a CdA, one does see flow with a CdA calculated from a Cd of 1.0 substantially above ASME and Cunningham.

There is a similarity between Driskell,Ward-Smith, Cunningham and Stanton in their description of the vena contracta occurring downstream of the orifice and moving toward the orifice as downstream pressure is reduced. Differences occur in the maximum value of C_d and/or value of Y. Ward-Smith indicates a C_d of 1.0 (for a thin orifice), but as noted above, using this in AFT Arrow produces a choked flow well above ASME and Cunningham. The accuracy of C/C_d calculated using ASME below pressure ratios of 0.75 is unknown.

To answer the question at the beginning of this paper, 'what CdA value to use for modeling choked flow through an orifice', that depends on the goal. If the goal is to insure a minimum flow rate, then one would want to use a low CdA value. On the other hand, if the goal is to identify a maximum possible flow, then a high CdA value would be used. From the comparative results presented above, low and high would correspond to a CdA calculated as the orifice area multiplied by a Cd value of 0.6 to 1.0. Note that these results represent only one possible fluid and upstream conditions. For a better approximation one may calculate the flow using the ASME of Cunningham equations and iteratively determine an appropriate Cd value for the specific conditions.

Further, if one needs to more precisely predict choked flow, then the system design should incorporate a nozzle in lieu of an orifice. Properly designed nozzles achieve a Cd under subsonic and choked flow conditions approaching one, that is, they approach the conditions of end point choking as one would see at the discharge from a pipe.

Finally, the last graph above is presented to illustrate the necessity to specify CdA values consistent with the Cd value with this orifice model. In this graph various Cd values are used but no CdA is specified causing a discontinuous transition from subsonic to choked flow as AFT Arrow moves from the subsonic calculation to the sonic.

References

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3) *Flow of Fluids, Crane Technical Paper No. 410*, © 1988 Crane Co. (Twenty Fifth Printing - 1991)

4) *Orifice Meters With Supercritical Compressible Flow*, R.G. Cunningham, Transactions of the ASME, July, 1951.

5) *IDEAL GAS FLOW THROUGH NOZZLES, RESTRICTIONS AND ENLARGEMENTS*, http://www.isa.org/books/Mulley_Papers/Nozzles.html

6) *Compressible Flow Analysis of Steam*, http://www.engsoft.co.kr/download_e/steam_flow_e.htm#4.%20Compressible%20Flow%20Analysis%20of%20Ideal%20Gas, ENGSoft Inc.

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And by reference from the above - "Critical flowmetering: The characteristics of cylindrical nozzles with sharp upstream edges", Ward-Smith, A.J., Int. J. Heat & Fluid Flow, vol 1, No 3, 1979

8) *Measurement of Fluid Flow in Pipes Using Orifice, Nozzle and Venturi*, ASME MFC-3M-1989, © 1990 by The American Society Of Mechanical Engineers.

9) AFT Arrow User's Guide, AFT Arrow version 4.0 Compressible Pipe Flow Modeling, © 2006 Applied Flow Technology Corporation.